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Brief Review of One Dimensional Neighbor Balanced Designs Since 1967

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Abstract

Rees (1967) introduced neighbor designs in serology. Since then it is well investigated field. Neighbor balanced designs are more useful to remove the neighbor effects in experiments where the performance of a treatment is affected by the treatments applied to its neighboring plots. Neighbor designs ensure that treatment comparisons will be less affected by neighbor effects as possible. This paper will provide an overview of the most important developments in this field.

Keywords: Neighbor designs; Generalized neighbor designs; Second order neighbor designs; All order neighbor balanced designs.

1. Introduction

1.1 Neighbor Balanced Designs

Rees (1967) introduced neighbor designs. Neighbor design is a collection of circular blocks in which any two treatments appear as neighbors equally often. A block is circular if the treatments allotted to its first and last plots are considered as neighbors. In circular block, each treatment has one left and other right neighbor. If no treatment occurs in a block more than once then block is called binary. A block is linear if it is formed in a line and the treatments allotted to its first and last plots are not considered as neighbors. A collection of linear blocks in which any two treatments appear as neighbors equally often is called equineighbored designs by Kiefer and Wynn (1981) but by Ipinyomi (1985) a design is equineighbored which is neighbor balanced at each order. The designs are called neighbor balanced at distance 2 by Azaiz *et al.* (1993) which have the property that for each ordered pair of distinct treatments there is exactly one plot that has the first chosen treatment as left neighbor and second chosen treatment as right neighbor. Rosa and Huang (1975) defined a balanced circuit design with restriction $k \leq v$ as: A balanced circuit design *BCD* (v, k, λ) with parameters v, b, r, k, λ' is an arrangement of v elements into b circular blocks such that each circular block contains k elements, each element occurs in exactly r circular blocks and any two distinct elements are linked in exactly λ'

circular blocks. Preece (1994) discussed balanced ouchterlony neighbor designs (BONDs). A BOND is an arrangement where the numbers of a set S of *v* distinct elements are applied in b blocks such that (i) each block contains k elements (k >2) drawn from S but are necessarily all distinct, (ii) the elements in each block are arranged on the circumference of a circle so that each of these element has two neighbors, (iii) each member of S appears exactly *r* times throughout the arrangement, (iv) no element of S ever has itself as a neighbor and (v) every element of S has each member of S as a neighbor exactly λ' times. According to Preece (1994), a Quasi Rees neighbor design is one that satisfies conditions (i) – (iv) of BOND and modified condition (v) as (v*) which is: every element from S has each other element as a neighbor exactly once except that it has just one of the other elements as a neighbor exactly twice.

1.2 Application of the neighbor designs

Neighbor designs were initially used in serology. Rees (1967) presented a technique used in virus research which requires the arrangement in circles of samples from a number of virus preparations such that over the whole set, a sample from each virus preparation appears next to a sample from every other virus preparation. In agro forestry intercropping experiments as trees are much taller than the crop, there is a neighbor effect through interplant competition. In this situation neighbor balance between the treatments must be looked for (see Monod, 1992). Experiments in agriculture, horticulture and forestry often show neighbor effects. In plants with an important root system, such as potatoes, varieties which germinate earlier will establish their roots and take nutrients from adjoining plots on both sides. In cereal crops or sunflowers, tall varieties may shade the plot on their North side. By Jenkyn and Dyke (1985), in pesticide or fungicide experiments, parts of the treatment may spread to the plot immediately downwind. In such situation neighbor balanced designs are useful to remove the neighbor effects.

2. One Dimensional Neighbor Models

In one dimensional designs, the blocks are well separated such that the observations from treatments allocated in different blocks are uncorrelated. Suppose, if we have a neighbor design for *v* treatments in b blocks each of size k, where the blocks are well separated, the following additive model can be considered for analysis.

$$\mathbf{Y} = \mathbf{1}\mu + \mathbf{X}_1\boldsymbol{\tau} + \mathbf{X}_2\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

where μ is the overall mean, $\boldsymbol{\tau}$ is vector of treatment effects of size $v\times 1$, $\boldsymbol{\beta}$ is vector of block effects of size $\mathrm{b}\times 1$, $\mathbf{X_1}$ is the incidence matrix for treatments, $\mathbf{X_2}$ is the incidence matrix for blocks and $\boldsymbol{\varepsilon}$ is a vector of random errors of size $\mathrm{n}\times 1$ with the assumptions E($\boldsymbol{\varepsilon}$) = 0 and Var($\boldsymbol{\varepsilon}$) = $\sigma^2\mathbf{V}$, where **V** is some symmetric positive definite matrix**.** When the observations are uncorrelated, then $\sigma^2\mathbf{V}=\sigma^2\mathbf{I}$. For correlated observations/neighbor effects following structures of **V** are suggested in literature, viz.

2.1 NN covariance model of Kiefer and Wynn (1981)

Kiefer and Wynn (1981) suggested the nearest neighbor (NN) covariance model (when the neighbor effect exists among the adjacent observations only) as:

$$Cov(y_{i,j}, y_{i',j'}) = \begin{cases} 1 & if \quad i=i' \ and \ j=j' \\ \rho & if \quad i=i' \ and \ |j-j'|=1 \end{cases}$$

It can also be written as $\mathbf{V} = I_b \otimes W_k$ (2.1)

Where $\otimes$ is kronecker product and W is a k$\times$k matrix such that

$$W_k = \begin{bmatrix} 1 & \rho & 0 & \cdots & 0 & 0 \\ \rho & 1 & \rho & \cdots & 0 & 0 \\ 0 & \rho & 1 & \cdots & 0 & 0 \\ 0 & 0 & \rho & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & \rho \\ 0 & 0 & 0 & \cdots & \rho & 1 \end{bmatrix}$$

The above model is useful when the neighbor effect exists among the adjacent neighbors only. When the neighbor effect exists among the adjacent observations as well as higher order neighbors then following models are useful.

2.2 NN covariance model of Patterson and Hunter (1983)

Patterson and Hunter (1983) suggested a model with a covariance matrix **V** such that

$$\mathbf{V} = I_n + \phi \ H_n \qquad (2.2)$$

Where

$$H_n = \begin{bmatrix} 0 & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & 0 & \rho & \cdots & \rho^{n-2} \\ \rho^2 & \rho & 0 & \cdots & \rho^{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \cdots & 0 \end{bmatrix}$$

This model is called the exponential variance model because the variance of the difference between the observations i and $i+\ell$ is 2(1-$\phi\,\rho^{\ell}$) which is the exponential function and ℓ is order of the neighbors. It has two parameters ϕ and ρ, where $0 \le \phi, \ \rho \le 1$.

2.3 Linear variance model of Williams (1987)

Williams (1987) suggested a linear variance model with a covariance matrix **V** such that

$$\mathbf{V} = I_n - \theta \ L_n \qquad (2.3)$$

Where

$$L_n = \begin{bmatrix} 0 & 1 & 2 & \cdots & n-1 \\ 1 & 0 & 1 & \cdots & n-2 \\ 2 & 1 & 0 & \cdots & n-3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ n-1 & n-2 & n-3 & \cdots & 0 \end{bmatrix}$$

This model is called the linear variance model because the variance of the difference between the observations i and $i+\ell$ is 2(1 $+\theta\ell$) which is the linear function.

2.4 NN covariance model of Ipinyomi (1985)

Ipinyomi (1985) suggested a more general model by assuming that correlation diminishes as the distance between the treatments (order of the neighbors) increases, that is, $\rho_1 \ge \rho_2 \ge \rho_3 \ge ... \ge \rho_{k-1} \ge 0$. ρ_ℓ is the correlation between any two observations allocated to plots j and $j+\ell$ in block i, where ℓ = 1, 2, …, k-1. The model with a covariance matrix **V** is $\mathbf{V} = I_b \otimes V_k$ (2.4)

Where $$V_k = \begin{bmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{k-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{k-2} \\ \rho_2 & \rho_1 & 1 & \cdots & \rho_{k-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \rho_{k-1} & \rho_{k-2} & \rho_{k-3} & \cdots & 1 \end{bmatrix}$$

2.5 Discussion on models presented in section 2.

Model 2.4 is more general because all models mentioned above are its special cases. By considering $\rho_2 = \rho_3 = ... = \rho_{k-1} = 0$ and $\rho_1 = \rho$, this model becomes model (2.1). If $\phi = 1$ and $\rho_\ell = \rho^\ell$, this model becomes Patterson and Hunter (1983) model and similarly by setting $\rho_\ell = \theta\ell$, this model becomes linear variance model suggested by Williams (1986).

3. One Dimensional Neighbor Designs in Circular Blocks

Following are the one dimensional neighbor balanced designs in circular blocks, already existed in literature.

3.1 Neighbor Designs Constructed by Rees (1967)

i) If $v = 2m + 1$ labeled as 0, 1, 2, …, v -1 then neighbor designs can be constructed by developing the following initial block cyclically mod 2*m*.

(0,1, -1, 2, -2, … , -(*m* -1), *m*, ∞), where ∞ = 2*m*.

ii) If v = 4t+3 is a power of a prime then a primitive root x exists such that $x^{2t+1} \equiv -1$ mod (4t+3) then required design is obtained by developing the following initial block cyclically mod v.

$(x^0, x^2, x^4, \ldots, x^{4t})$

iii) Rees (1967) also listed neighbor designs for every v up to 41 with k ≤ 10 and $\lambda' = 1$.

3.2 Neighbor Designs by Hwang (1973)

Following are some infinite series of neighbor designs which are constructed by Hwang (1973).

Series (i). **For $v = 2k + 1$, k > 2 and $\lambda' = 1$**

Let $F_k(1) = (f_1, f_2, ..., f_k)$ denote the sequence of forward differences in the initial block and 'C' a constant. Let $F_k(1) \circ C$ denote the sequence $(f_1 \circ C, f_2 \circ C, ..., f_k \circ C)$, where

$$f_i \circ C = f_i + C \quad \text{if} \quad f_i \geq 0$$
$$f_i \circ C = f_i - C \quad \text{if} \quad f_i < 0 \qquad (\text{mod } v)$$

For k = 3, let $F_3(1)$ = (1, 2, -3),

For k = 4, let $F_4(1)$ = (1, -2, -3, 4),

For k = 5, let $F_5(1)$ = (1, -2, 3, 4, -6),

For k = 6, let $F_6(1)$ = (1, -2, 3, - 4, -5, 7)

Suppose F_3 (1), F_4 (1), …, F_{k-1}(1) are given then F_k(1) can be constructed, in general, for k ≥ 7 by defining

$$F_k(1) = (1, -2, -3, 4, F_{k-4}(1) \circ 4) \qquad \text{if k is even,}$$

F_k (1) = (1, 2, -3, $F_{k-3}(1) \circ 3$) if k is odd.

Series (ii). **For $v = 2mk + 1$, $k \equiv 0 \pmod 2$, $k > 2$, $\lambda' = 1$ with *m* initial blocks**

Let $G_k(m) = \{F_k(1), F_k(2), \ldots, F_k(m)\}$ where $F_k(i)$ denotes the *i*th initial block. To specify $F_k(i)$, put $v = 2^j k + 1$ (*j* is an arbitrary integer) and then

$$F_k(i) = F_k(1) \circ 2(i-1)k, \quad i = 1, 2, 3, \ldots, 2^{j-1}.$$

Series (iii). **For $v = 2mk+1$, $k \equiv 0 \pmod 4$, $\lambda' = 1$ with *m* initial blocks**

Let $G_k(m) = \{F_k(1), F_k(2), \ldots, F_k(m)\}$ where $F_k(i)$ denotes the *i*th initial block

$F_k(i) = \{\varepsilon_y(k \times i - k + y): y = 1, 2, \ldots, k\}$, $i = 1, 2, \ldots, m$,

$$\varepsilon_y = 1, \quad \text{if } y \equiv 0, 1 \pmod 4,$$
$$\varepsilon_y = -1, \quad \text{if } y \equiv 2, 3 \pmod 4.$$

3.3 Neighbor Designs by Bermond and Faber (1976).

Bermond and Faber (1976) constructed neighbor designs for the following cases.

Case (i). Let $v = 2m$, *m* odd and define the length of the directed edge (x, y) to be y-x (mod *v*). We construct three closed paths from the edges of lengths ± *m* and ± 1:

$$P_1 = (0, 1, m+1, m+2, 2, 3, m+3, m+4, 4, 5, \ldots, m-1, m)$$
$$P_2 = (0, 1, m, m+1, 1, 2, m+2, m+3, 3, \ldots, m-1, 2m-1)$$
$$P_3 = (0, 2m-1, 2m-2, \ldots, 2, 1)$$

If $1 < x < m$, we construct two paths from edges of lengths x and $y = m+1-x$, where $x = (m+1)/2$:

$$P_x = (0, x, m+1, x+m+1, 2(m+1), \ldots, x+(2m-1)(m+1));$$
$$P_y = (0, y, m+1, y+m+1, 2(m+1), \ldots, y+(2m-1)(m+1)).$$

Since ged $(m+1, 2m) = 2$, each of these paths has length $2m$, and there are $m = 3$ of them. If $m < x < 2m-1$, we construct another m-3 paths similarly, where $x + y = -(m+1)$ and $x = -(m+1)/2$. Finally we take the path

$$P_{(m+1)/2} = (0, (m+1)/2, m+1, 3(m+1)/2, \ldots, (2m-1)(m+1)/2)$$

Similarly $P_{-(m+1)/2}$. Since ged $((m+1)/2, 2m) = 1$, each of these two paths has length $2m$. This gives $3 + 2(m-3) + 2 = 2m-1$ paths as required which provide a neighbor design with $v \equiv 2 \pmod 4$, $v = k$, $b = v-1 = r$ and $\lambda' = 2$.

Case (ii). For $k = v-1$, *v* even, $b = v$ and $\lambda' = 2$ consider the following initial block.

$$C_0 = (\infty, 0, k-1, 1, k-2, \ldots, [k/4]-1, [3k/4]+1, [k/4], [3k/4]+1, \ldots, [k/2]-1, [k/2]+1)$$

If $C_i = C_0 + i \pmod k$, the set of circuits $C_0, C_1, \ldots, C_{k-1}$ together with the circuit

$$C_\infty = (0, h, 2h, \ldots, (k-1)h) \bmod k, \text{ where } [k/2]+1 = h.$$

3.4 Neighbor Designs by Hwang and Lin (1977)

(i) Let $v = 2n+1 = p\,t$, $k = p$ and odd prime, M be an $m \times k$, 2-balanced array mod *v* whose elements are precisely the set (I_n - S), where S = {t, 2t, … , ((*p*-1)/2)t} and $[B_1]$, $[B_2]$, …, $[B_m]$ be *m* sets of blocks from M. For each $j = 1, 2, \ldots,$

(p-1)/2, we construct a set of t blocks $[A_j] = \{A_{j1}, A_{j2}, \ldots, A_{jt}\}$ where $A_{ji} = (i, jt+i, 2jt+i, \ldots, (p-1)jt+i)$

(ii) Let v = 2n+2, k = p and odd prime, v = pt, S = { t, 2t, … , ((p-1)/2)t}. If (I_n - S) can be arranged into a 2-balanced array mod v*, we can construct ND (v, k, 2).

3.5 Dey and Chakravarty (1977)

(i) If v = 6t+1 be a prime power then there exist a neighbor design with v = 6t +1, b = t(6t +1), r = 3t, k = 3, λ' = 1.

(ii) If v = 6t+3 then there exists a neighbor design for k = 3 with r = 3t+1 and λ' = 1 in b = (3t+1)(2t +1).

(iii) Let v = 4t+3 be a prime power, x a primitive element of GF (v), $x^0 + x = x^{2u}$ (an even power of x) then following initial blocks provide neighbor designs for k = 3 with r = 3(2t +1) and λ' = 3 in b = (2t +1)(4t +3).

$$(0,\ x^i,\ x^{2u+i}), \qquad i = 0, 2, 4, \ldots, 4t.$$

(iv) If v = 4t+3 be a prime power and x a primitive element of GF (v), $x^0 + x = x^{2m+1}$ (an even power of x) then following initial blocks provide neighbor designs for k = 3 with r = 3(2t +1) and λ' = 3 in b = (2t +1)(4t +3).

$$(0,\ x^{i+1},\ x^{2m+1+i}), \qquad i = 0, 2, 4, \ldots, 4t.$$

(v) If v = 4t+3 be a prime power and x a primitive element of GF (v) then following initial blocks provide neighbor designs for k = 3 with λ' = 4 and r = 4(2t +1) in b = (2t +1)(4t +3)

$$(0,\ x^{2u},\ 0,\ x^{2u+2}), \quad u = 0, 1, \ldots, 2t.$$

(vi) If v = 4t+3 be a prime power and x a primitive element of GF (v) then following (2t+1) initial blocks provide neighbor design for k = 2t+2 with r = (2t +1)(2t +2) and λ' = 2t+2 in b = (2t +1)(4t +3).

$$(x^0, x^2, x^0, x^4, x^6, x^8, \ldots, x^{4t}),$$
$$(x^0, x^2, x^4, x^2, x^6, x^8, \ldots, x^{4t})$$
$$(x^0, x^2, x^4, x^6, x^4, x^8, \ldots, x^{4t}),$$
$$\ldots$$
$$(x^0, x^2, x^4, x^6, \ldots, x^{4t-2}, x^{4t}, x^{4t-2}),$$
$$(x^{4t}, x^2, x^4, x^6, \ldots, x^{4t}, x^0)$$

(vii) If v = 2t+3 be a prime power and x a primitive element of GF (v) then following initial blocks provide neighbor designs for k = 4t+2 with r = 4t+2 and λ' = 2 in b = 4t+3.

$$(0,\ x^0,\ 0,\ x^2,\ \ldots,\ 0,\ x^{4t})$$

(viii) If v = 2t+1 be a prime power and x a primitive element of GF (v) then following initial blocks provide neighbor designs for k = 4t with r = 8t and λ' = 4 in b = 8t+2,

$$(0,\ x^0,\ 0,\ x^2,\ \ldots,\ 0,\ x^{4t-2}) \text{ and } (0,\ x,\ 0,\ x^3,\ \ldots,\ 0,\ x^{4t-4})$$

3.6 Neighbor designs by Chandak (1981)

- Let v be a prime or power of prime, x be the primitive of Galois field GF (v), C be the multiplicative group of GF (v) and $m \geq 2$ divide v-1, say m = (v-1)/s. Then x^m has order s. A neighbor design with parameters v, b = mv, r = (v-1), k = s and λ' = 2 can be obtained by developing the cosets $C_1, C_2, ..., C_m$ of the factor subgroup C/C_1 mod v.
- If v = 4t+3 a prime power and x a primitive element of GF(v) then following t initial blocks provide neighbor designs for k = 2t+1 with b = t(4t+3) and r = t(2t +1)

$$(x^0,\ x^2,\ x^4, \dots,\ x^{4t})$$
$$(x^0, x^4,\ x^8, \dots,\ x^{4t-2})$$
$$(x^0,\ x^6,\ x^{12}, \dots,\ x^{4t-4})$$
$$\dots$$
$$(x^0,\ x^{2t},\ x^{4t}, \dots,\ x^{2t+2})$$

- He also constructed neighbor designs with parameters $v = s^2$, b = $s^2(s+1)$, $r = 2(s^2-1)$, k = 2(s -1), λ' = 4 through Euclidean Geometry.

3.7 Neighbor designs by Metti (1996)

Neighbor designs for v = 2m and k = m can be constructed in 2(v-1) blocks developing the following two initial blocks cyclically mod (2m+1).

S = $\{a_0, a_1, a_2, ..., a_{(m+1)/2}, c_1, c_2, ..., c_{(m-1)/2}\}$ and S* = $\{0, x_1, x_2, ..., x_{m-1}, \infty\}$, where

(i) $a_i = \sum u_i$; $i = 0, 1, 2, ..., (m+1)/2$

(ii) $c_j = a_{(m+1)/2-j}$; $j = 0, 1, 2, ..., (m-1)/2$

(iii) $x_i = i(m+1)/2 + a_i$; $i = 0, 1, 2, ..., (m-1)/2$

(iv) $x_j = x_{(m-1)-j}$; $j = (m+1)/2, (m+3)/2, ..., (m-1)$

3.8 Neighbor designs by Ahmed and Akhtar (2008a, b)

(i) If v = 4s; s a natural number and m = 2s-1 then neighbor designs are obtained by developing the following initial block cyclically mod 2m.

(0, 1, 3, 6, … , (m-1)m/2, m (m+1)/2, m(m+3)/2, … , m(m+1) -1, ∞) mod (2m+1)

(ii) If v = 2m+1 and k = v-1 then neighbor designs can be obtained by developing the following initial block cyclically mod 2m with augmenting the block ((v-2), (v-3),…, 2, 1, 0).

(0, 1, -1, 2, -2, … , -(m-1), ∞) mod 2m, where ∞ = 2m.

(iii) If v = 4s, k = v-1 and m = 2s-1 then neighbor designs can be obtained by developing the following initial block cyclically mod 2m with augmenting the block (0, 1, 2, … , k-1).

(0,1,3,6,…, (m-1)m/2, m (m+1)/2, m(m+3)/2, … ,m(m+1) -1, ∞) mod (2m+1)

(iv) Neighbor balanced designs for v = 2k through two initial blocks which are binary.

(v) They also presented some more neighbor balanced designs.

Iqbal *et al.* (2009) listed neighbor balanced designs which are constructed using method of cyclic shifts.

3.9 Discussion on Neighbor designs presented in Section 3.

Rees (1967) gave the simple method to construct neighbor designs for odd v when k = v. He also presented a list of minimal neighbor designs for odd v up to 41 and k ≤ 10. Hwang (1973), Dey and Chakravarty (1977), Hwang and Lin (1977) and Chandak (1981) gave several series of neighbor designs for odd v which is a remarkable contribution in this area but these designs are not necessarily binary. Bermond & Faber (1976), and Hwang & Lin (1977) developed some series of these designs for even v. Ahmed and Akhtar (2008a, b) presented several series of neighbor designs in binary circular blocks in which their construction is quite easy.

4. Neighbor Designs in Linear Blocks

4.1 Neighbor Designs by Kiefer and Wynn (1981) for k = v

Kiefer and Wynn (1981) introduced an algorithm to construct the complete block neighbor designs in linear blocks.

- The first $v/2$ rows of **A** constitute a complete block neighbor design when v is even.
- The v rows of **A** constitute a complete block neighbor design when v is odd.

Where **A** is a $v \times v$ square matrix whose (j, l)th cell ($1 \le j, l \le v$) is given by

$$a_{jl} = [\sum_{r=1}^{j}(-1)^r (r-1) + \sum_{r=1}^{l} (-1)^r (r-1)] \mod v \qquad (3.1)$$

4.2 Neighbor designs in Linear Blocks by Cheng (1983)

i) For v odd and k = 3: Choose the BIBD with k = 2 and b = $v(v$-1)/2 i.e. all the possible pairs of v treatments. Partition the $v(v$-1)/2 blocks of size two into v groups $B_1, B_2, ..., B_v$ of (v-1)/2 blocks such that for each i, the ith treatment does not appear in B_i and each other treatment appears in B_i exactly once. By inserting treatment i in the middle of each block, we get required design.

ii) For v even and k = 3: Take two copies of the BIBD with k = 2 and b = $v(v$-1) and partition the $v(v$-1)/2 blocks of size two into v groups $B_1, B_2, ..., B_v$ of (v-1) blocks such that for each i, the ith treatment does not appear in B_i and each other treatment appears in B_i exactly twice. By inserting treatment i in the middle of each block, we get required design.

iii) For v odd and k = v-2: Let $v = 2m+1$ and C_i be the Hamiltonian cycle (0, i, i+1, i-1, i+2,…, $i+m$, 0), where all the components except 0 are taken as the positive integers 1, 2,…, $2m$ mod $2m$. From each of these Hamiltonian cycles, construct v linear blocks of size v-2 by the method of cyclic permutation, we get required design with $v(v$-1)/2 blocks of size v-2.

iv) For *v* even and k = *v*-2: A block of size *v*-2 can be obtained by deleting any of such pairs while keeping the order of the other treatments of **A** developed from (3.1). The *v*(*v*-1)/2 blocks of size *v*-2 thus constructed from the first *v*/2 rows of **A** constitute the required design.

v) For *v* odd and k= *v* -1: Let C_i be the Hamiltonian cycle (0, *i*, *i*+1, *i*-1, *i*+2,…, *i*+*m*, 0) for each *i*, $1 \le i \le m$, where all the components except 0 are taken as the positive integers mod 2*m*. Each C_i can be considered as a circular block of size *v*. From each C_i we can construct *v* linear blocks of size *v*-1 by cyclic permutation. This yields our required design with b = *v*(*v*-1)/2 and k = *v*-1.

4.3 Neighbor Designs in Linear Blocks of Size 3 by Jacroux (1998)

Jacroux (1998) constructed neighbor designs in linear blocks of size 3 for all *v* which are efficient under standard intrablock analysis as well as when experimental units adjacent within blocks are correlated.

Case (i). If $v \equiv 1$ (mod 4) then the following (*v*-1)/4 initial blocks provide the equineighbored designs in linear blocks of size 3.

(0, *i*, 2*i*+(*v*-1)/2); *i* = 1,2, ..., (*v*-1)/4.

Case (ii). If $v \equiv 3$ (mod 4) then the following (*v*-1)/2 initial blocks provide the equi-neighbored designs in linear blocks of size 3.

(i) (0, *i*, 2*i* +1); *i* = 1, …, (*v*-3)/2, $i \neq (v\text{-}3)/4$,

(ii) (0, (*v*-3)/4, *v*-1) and (0, (*v*-1)/2, (*v*+1)/2).

Case (iii). If $v \equiv 0$ (mod 4) then consider *v*/4 initial blocks (0, *i*, (*v*/2)+2*i*-1); *i*= 2, …, *v*/4. Generate *v*(*v*-4)/4 blocks cyclically from these initial blocks and then augment these resultant blocks with those given by

(i) (*j*-1, *j*, (*v*/2)+ *j*) for *j* = 1, …, *v*/2 and

(ii) (((*v*/2)-2)+2*j*, ((*v*/2)-1)+2*j*, (*v*/2)+2*j*) for *j* = 1, …, *v*/4.

Case (iv). If $v \equiv 2$ (mod 4) then consider (*v*-2)/2 initial blocks (0, *i*, 2*i*+1); *i*= 1, ..., (*v*-2)/2. Generate *v*(*v*-2)/2 blocks cyclically from these initial blocks and then augment these resultant blocks with those given by (2*j*-2, 2*j*-1, 2*j*) for *j* = 1, …, *v*/2.

4.4 Neighbor Designs in Linear Blocks by Ahmed and Akhtar (2010)

(i) NBD can be generated with parameters *v* = 4*i*+1, *i* integer, k = 3, $\lambda' = 1$, λ = 1 or 2 and b = *iv* from the following *i* sets of shifts.

S_j = [2*j*-1, 2*j*]; *j* = 1, …, *i*.

(ii) NBD can be generated with parameters *v* = 2*i*+1, *i* (>1) odd, k = 3, $\lambda' = 2$, $\lambda = 3$ and b = *iv* from the following *i* sets of shifts.

S_j = [*j*, *j*]; *j* = 1, …, *i*.

(iii) NBD can be generated with parameters *v* = 4*i*-1, *i* (>1) integer, $k_1 = 3$, $k_2 = 2$, $\lambda' = 1$, λ = 1 or 2, b_1 = (*i*-1)*v* and $b_2 = v$ from the following *i* sets of shifts. S_j = [p, p+1]; *j* = 1, …, *i*-1 and p = 2*j*-1. S_i =[(*v*-1)/2]

(iv) NBD can be generated with parameters *v* = 4*i*, *i* (>1) integer, $k_1 = 3$, $k_2 = 2$, $\lambda' = 1$, λ = 1 or 2, b_1 = (*i*-1)*v* and b_2 = 3*v*/2 from the following (*i*+1) sets of shifts.

S_j =[p, p+1]; j = 1, …, i-1 and p = 2j-1. S_i =[(v-2)/2] and S_{i+1} = [v/2](1/2)

(v) NBD can be generated with parameters v = 2i+1 and prime**,** k < v, λ' = k-1, $\lambda = \sum_{u=1}^{k-1} u$ and b = iv from the following i sets of shifts.

$$S_j = [j, j, \ldots, j]; \quad j = 1, \ldots, i.$$

4.5 Discussion on Neighbor designs presented in Section 4.

Kiefer and Wynn (1981) introduced an algorithm to generate neighbor designs in linear blocks for every v = k. Cheng (1983) gave several methods but these constructions require a large number of blocks. Jacroux (1998) gave the complete solution of equineighbored for k = 3 in linear blocks. Most of the neighbor designs presented by Ahmed and Akhter (2010) in linear blocks are economical.

5. Generalized Neighbor Designs

Generalized t-neighbor design defined by Misra *et al*. (1991) is an arrangement of v treatments in b circular blocks such that (i) each treatment appears r times in the design (not necessarily in r distinct blocks), (ii) blocks have k_1, k_2,…, k_b treatments (same treatment should not occur side by side), (iii) any two treatments can occur as neighbor λ_1', λ_2',…, λ_t' times, and (iv) for a given treatment θ, there are n_i treatments which occur λ_i times as neighbor where the number n_i is independent of the treatment θ. They constructed GNDs for only v odd.

5.1 $\boldsymbol{GN_2}$ *-designs by Rees (1967) for v = k = 2m*

Rees (1967) suggested that a design for 2m antigens can be derived through the design for 2m+1 antigens, by deleting the (2m+1)th number, but in resultant design one comparison is duplicated per antigen. From this idea, the initial block is

(0, 1, -1, 2, -2, … , -(m-1), m) mod 2m.

Remaining (m-1) blocks are obtained by developing the initial block cyclically mod 2m.

5.2 GNDs constructed by Chaure and Misra (1996)

They constructed:

(i) **GNDs** for v = 4t +1, k = 3 in b = t(4t+1),

(ii) $\boldsymbol{GN_3}$ -designs for v = 4t, k = 2t in b = 2(4t -1), where t >2,

(iii) $\boldsymbol{GN_2}$ -designs for v = 4t-1, k = 2n+1, where 'n' is a positive integer,

(iv) $\boldsymbol{GN_2}$ -designs for v = 4t-1, k = 2n.

5.3 GNDs constructed by Nutan (2007)

Nutan (2007) developed a family of proper generalized neighbor designs and constructed GN_2 - designs for v = b =k =2m, λ_1' = 2, λ_2' = 4, n_1 = v-2 by developing the following initial block cyclically mod v. (0, v-1, 1, v-2, 2, …, v/2)

5.4 Generalized neighbor designs by Kedia and Misra (2008)

They constructed:

(i) GN_2 - designs for v = 3t+1, t being any positive integer, b = vt, r = 4t, k = 4, n_1 = 2t, n_2 = t, λ_1' = 3 and λ_2' = 2 by developing the following initial blocks mod v. B_i = (i, 0, 2t + i, t); i = 1, 2, 3, …,t -1; B_t = (0, 3t, t, 2t).

(ii) GN_2 - designs for v = 5t+1, t being any positive integer, b = vt, r = 4t, k = 4, n_1 = 2t, n_2 = 3t, λ_1' = 1 and λ_2' = 2 by developing the following initial blocks mod v. B_i = (0, i, 3t, 4t + i); i = 1, 2, 3, …,t-1; B_t = (0, 4t, 2t, 3t).

(iii) GN_3 - designs for v = 5t+1, t being any positive integer, b = vt, r = 4t, k = 4, n_1 = 2t+2, n_2 = 3t-4, n_3 = 2 λ_1' = 1, λ_2' = 2 and λ_3' = 3 by developing the following initial block mod v. B_i = (i, 0, t + i, 4t); i = 1, 2, t.

(iv) GN_2 - designs for v = 6t+1, t being any positive integer, b = vt, r = 4t, k = 4, n_1 = 4t, n_2 = 2t, λ_1' = 1 and λ_2' = 2 by developing the following initial block mod v. B_i = (t - i, 5t, 2t + i, t); i = 1, 2,…, t.

(v) GN_3 - designs for v = 6t+1, t being any positive integer, b = vt, r = 6t, k = 6, n_1 = 2t -2, n_2 = 2t +4, n_3 = 2t-2 λ_1' = 1, λ_2' = 2 and λ_3' = 3 by developing the following initial block mod v. B_i = (2t + i, 0, i, 2t, 5t + i, t); i = 1, 2, t.

(vi) GN_2 - designs for v = 7t+1, t being any positive integer, b = vt, r = 6t, k = 6, n_1 = 5t, n_2 = 2t, λ_1' = 2 and λ_2' = 1 by developing the following initial block mod v. B_i = (2t + i, 0, i, 2t, 5t + i, t); i = 1, 2,…, t.

5.5 ***GN_2*** *-designs by Ahmed et al. (2009)*

(i) GN_2 - designs can be constructed with parameters v = 2t+1, k = t+1, n_1 = 2t -2, n_2 = 2, λ_1' =1 and λ_2' =2, where t >1 is an integer, in v circular blocks from the set of shifts. S = [1, 2, … , t -1, t]

(ii) GN_2 - designs can be constructed with parameters v = 2t+1, k = t+2, n_1 = 2t-4, n_2 = 4, λ_1' =1 and λ_2' =2, where t >2 is an integer, in v circular blocks from the set of cyclic shifts. S_1 = [1, 2, … , t, α], where α is any value among 1, 2,…, t-1 such that (1 + 2 + … + t + α) mod v ≠ 0, α, v- α.

(iii) GN_2 - designs can be constructed with parameters v = 2t+1, k = t+3, n_1 = 2t- 6, n_2 = 6, λ_1' =1 and λ_2' =2, where t >3 is an integer, in v circular blocks from the set of cyclic shifts. S_2 = [1, 2, … t-1, t, t-1, α], where α is any value among 1, 2, … , t-2 such that (1 + 2 + … + t + α) mod v ≠ 0, α, v- α, t-1, t+2.

(iv) GN_2 -designs can be constructed with parameters v = 2t, k = t+1, r_1 = t, r_2 = v-1, n_1 = 2t- 2, n_2 = 1, λ_1' =1 and λ_2' =2, where t >2 is an integer, in v-1 circular blocks from the set of cyclic shifts. S_3 = [1, 2, … , t -1]t

(v) GN_2 -design can be constructed with parameters v = 2t, k = t+2, r_1 = t, r_2 = v-1, n_1 = 2t- 4, n_2 = 3, λ_1' =1 and λ_2' =2, where t >3 is an integer, in v-1 circular blocks from the set of cyclic shifts. S_4 = [1, 2, … t-1, t-1]t

(vi) GN_2 -design can be constructed with parameters v = 2t, k = t+3, r_1 = t, r_2 = v-1, n_1 = 2t- 6, n_2 = 5, λ_1' =1 and λ_2' = 2, where t >4 be an integer, in v-1 circular blocks from the set of cyclic shifts. S_5 = [1, 2, … , t-1, t-1, t-2]t

5.6 ***GN_2*** *-designs by Zafaryab et al. (2010)*

(i) If v = 8t+2, t is positive integer and k = 4 then following (t+1) sets provide GN_2-design with parameters λ_1' =1 and λ_2' =2, n_1 = v-8, n_2 = 7 and b = v(t+1).
S_{j+1}= [v-(4j+1), 4j+2, 4j+3]; j = 0, 1, 2, …, t-1. S_{t+1} = [v/2, 2, 4]

(ii) If $v = 8t+3$, t is positive integer and k = 4 then following (t+1) sets provide GN_2-design with parameters $\lambda_1' = 1$ and $\lambda_2' = 2$, $n_1 = v-7$, $n_2 = 6$ and $b = v(t+1)$.

$S_{j+1} = [v-(4j+1), 4j+2, 4j+3]$; $j = 0, 1, 2, \ldots, t-1$. $S_{t+1} = [(v-1)/2, 1, 2]$

(iii) If $v = 8t+4$, t is positive integer and k = 4 then following (t+1) sets provide GN_2-design with parameters $\lambda_1' = 1$ and $\lambda_2' = 2$, $n_1 = v-6$, $n_2 = 5$ and $b = v(t+1)$.

$S_{j+1} = [v-(4j+1), 4j+2, 4j+3]$; $j = 0, 1, 2, \ldots, t-1$. $S_{t+1} = [(v-2)/2, v/2, 2]$

(iv) If $v = 8t+5$, t is positive integer and k = 4 then following (t+1) sets provide GN_2-design with parameters $\lambda_1' = 1$ and $\lambda_2' = 2$, $n_1 = v-5$, $n_2 = 4$ and $b = v(t+1)$.

$S_{j+1} = [v-(4j+1), 4j+2, 4j+3]$; $j = 0, 1, 2, \ldots, t-1$.$S_{t+1} = [(v-3)/2 , (v-1)/2, 3]$

(v) If $v = 8t+6$, t is positive integer and k = 4 then following (t+1) sets provide GN_2-design with parameters $\lambda_1' = 1$ and $\lambda_2' = 2$, $n_1 = v-4$, $n_2 = 3$ and $b = v(t+1)$.
$S_{j+1} = [v-(4j+1), 4j+2, 4j+3]$; $j = 0, 1, 2, \ldots, t$.

(vi) If $v = 8t+7$, $t \geq 0$ and k = 4 then following (t+1) sets provide GN_2-design with parameters $\lambda_1' = 1$ and $\lambda_2' = 2$, $n_1 = v-3$, $n_2 = 2$ and $b = v(t+1)$.

$S_{j+1} = [v-(4j+1), 4j+2, 4j+3]$; $j = 0, 1, 2, \ldots, t$.

(vii) If $v = 8t+8$, $t \geq 0$ and k = 4 then following (t+1) sets provide GN_2-design with parameters $\lambda_1' = 1$ and $\lambda_2' = 2$, $n_1 = v-2$, $n_2 = 1$ and $b = v(t+1)$.

$S_{j+1} = [v-(4j+1), 4j+2, 4j+3]$; $j = 0, 1, 2, \ldots, t$.

They also constructed GN_2-design for k = 6, 8, and 10 and listed these designs for k = 5, 7, and 9.

5.7 Discussion on Neighbor designs presented in Section 5

GN_2-designs constructed by Ahmed *et al*. (2009) are economical as compared with any of all others such as Misra *et al*. (1991), Chaure and Misra (1996), Nutan (2007), Kedia and Misra (2008). GN_2-designs constructed by Ahmed *et al*. (2009) are general for k. Zafaryab *et al*. (2010) also presented the economical GN_2-designs which are general for v when k is fixed such as 4, 6, 8, and 10.

6. Second and Higher Order Neighbor Balanced Designs

Keedwell (1984) considered 2-fold perfect circuit designs, these being balanced circuit designs whose neighbor properties apply not only to immediate neighbors but also to neighbors that are two places apart. Iqbal *et al*. (2006) constructed second order neighbor designs for circular blocks using method of cyclic shifts for $3 \leq k \leq 7$. Akhtar and Ahmed (2009) constructed several second and third order neighbor balanced designs for (i) v = k, (ii) k = v-1, (iii) v = 2k+1, and (iv) v = 2k, etc.

6.1 Equineighbored Designs by Ipinyomi (1985)

Ipinyomi (1985) constructed such designs with minimum blocks for $v = 2m+1$ and prime. He also constructed equineighbored designs of size

(i) (v, mkv, k), for v = 3k + 1 and v is a prime power and k is not even,

(ii) (v, mkv/2, k), for v = 3k + 1 and v is a prime power and k is even,

(iii) (v, kv, k), for v = 2k + 1 is prime,

(iv) (v, mv, 3), for $v = 2m + 1$,

(v) (v, mv, k), for $v = 2m + 1$ and v is prime, k < v and

(vi) (v, $v(v$-1), k), for k $\leq$ v by the columns of the k rows of a complete set of orthogonal Latin squares of order v.

6.2 All Order Neighbor Balanced Designs By Ai et al. (2007)

Ai *et al.* (2007) constructed all order neighbor balanced designs (ANBD) for v =2m+1 (odd prime) in 2m circular blocks through the following initial block.

$$B_i = \{(0, i, 2i, \ldots, (v\text{-}1)i) \bmod v, \ \ i = 1, 2, \ldots, v\text{-}1\}.$$

They also developed the methods of construction for all order balanced circular block neighbor designs when k < v and showed that the CNBD's (circular neighbor balanced design) exists under the following conditions.

(i) If v be an odd prime and $r \geq 1$ be an integer then there exists a CNBD (v^r, v^{r-1} (v^{r-1}-1), v; v-1).

(ii) If v = p^r be any prime power and k > 2 be such that k is a divisor of v-1 then there exists a CNBD (v, v (v -1)/k, k; k-1).

(iii) If q is a prime power and f is the greatest common divisor of q-1 and k then there exists a CNBD (q, q (q -1)/f, k; k-1).

(iv) If q = kn+1 is a prime power, there exists a CNBD (n, n(n -1), k; k-1).

6.3 All Order Neighbor Balanced Designs By Ahmed and Akhtar (2009)

Ahmed and Akhtar (2009) constructed following ANBD by using method of cyclic shifts.

(i) for v =2m+1and k = v prime through the following m sets of shifts each of k-1 elements with λ' =1.

$$Q_i = [i, i, \ldots, i](1/v) : i = 1, 2, \ldots, m.$$

(ii) for v =2m+1 (prime) through the following m sets of shifts each of k-1 elements with λ' = k.

$$Q_i = [i, i, \ldots, i] : i = 1, 2, \ldots, m.$$

6.4 Discussion on Neighbor designs presented in Section 6

Ipinyomi (1985) constructed equineighbored designs for almost all v in linear blocks while Ai *et al.* (2007) and Ahmed and Akhtar (2009) constructed all order neighbor balanced designs in circular blocks only for v prime or prime power. Second and higher order neighbor balanced designs presented by Akhtar and Ahmed (2009) are much useful to neutralize the neighbor effects.

7. Optimality of Neighbor Designs

Kiefer (1975) introduced the universal optimality criterion which includes the well known D, A and E optimality criteria as special case. Kunert (1984) showed that circular neighbor designs in $\Omega_{(v,v-1,v)}$ are universally optimal for the estimation of treatments as well as neighbor effects.

Druilhet (1999) constructed universally optimal design for models which incorporate one-sided or two-sided neighbor effects among the class of all equireplicated design or even more general classes and expressed that the link between block balance and neighbor balance is essential to prove optimality. Druilhet (1999) generalized the result of Kunert (1984) such that a circular neighbor balanced design d* is universally optimal for the estimation of treatment effects as well as neighbor effects over all designs in $\Omega_{(v,b,k)}$ for $3 \le k \le v$. He also showed that a circular neighbor balanced designs at distance 1 and 2 (CNBD2), d* is universally optimal for the estimation of treatment effects as well as neighbor effects over all designs in $\Omega_{(v,b,k)}$ which have no treatment preceded by itself, for $3 \le k \le v$. Further he proved that for $v = 5$ and $v \ge 7$ a CNBD2, d* is universally optimal for the estimation of treatment effects as well as neighbor effects over the class of equireplicated designs in $\Omega_{(v,v-1,v)}$. Druilhet (1999) also showed that for $v \ge 13$ a CNBD2, d* is universally optimal for the estimation of treatment effects as well as neighbor effects over the class of equireplicated designs in $\Omega_{(v,v,v-1)}$, where CNBD2 (Circular neighbor balanced designs at distance 1 and 2) is a circular neighbor balanced design where for each ordered pair of distinct treatments there exist exactly 'ℓ' plots that have the first chosen treatment as left neighbor and the second one as right neighbor with $\ell = bk/v(v-1)$ an integer and $k \le v$.

Bailey and Druilhet (2004) considered optimality of circular neighbor balanced block designs when neighbor effects are present in the model. They showed that circular neighbor balanced designs are universally optimal for total effects among designs with no self neighbor. They also showed some situations where a design with self neighbors is preferred to a neighbor balanced design.

Filipiak and Markiewicz (2005) studied optimality of circular neighbor balanced designs at distance 1 and 2 under the one-dimensional interference model. Filipiak and Rozanski (2005) showed that circular neighbor balanced designs are universally optimal. Ai. *et al*. (2007) generalized the results of Bailey and Druilhet (2004) to linear models containing the neighbor effects and showed that a circular block design neighbor balanced at distances up to $\gamma \le k - 1$ is universally optimal for total effects.

Jaggi *et al*. (2007) considered the following two cases of the designs D(v, b, k).

Case(i). The class of designs D_1 (v, b, k) which satisfy the following conditions.

(i) Each treatment appears in a given block an equal number of times, say λ (>1) times.

(ii) For each ordered pair of treatment (including identical pairs), there exists a constant number (λ_1') of plots that have the first chosen treatment as left neighbor and the second one as right neighbor.

For D_1 (v, b, k), they proved that a design d* $\in D_1$ (v, b, k) is universally optimal for the estimation of direct effects if $N_1 = N_2 = \lambda_1' J_V$.

Case (ii). The class of designs D_2 (v, b, k) which satisfy the following conditions.

(i) Each treatment appears in a given block an equal number of times, say, λ times.

(ii) Each ordered pair of treatment (excluding identical pairs), appear together as left and right neighbors a constant number of times (λ_1').

For D_2 (v, b, k), they proved that a design d* $\in D_2$ (v, b, k) is universally optimal for the estimation of direct effects if $\mathbf{N_L} = \mathbf{N_R} = \lambda_1' (J_V - I_V)$, where $\mathbf{N_L}$ is ($v \times v$) incidence matrix of direct versus left treatment, $\mathbf{N_R}$ is ($v \times v$) incidence matrix of direct versus right treatment respectively and $k = \lambda v$. In the next sections, universal optimal neighbor designs are considered for $k \leq v$.

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