Estimation of Parameters of the Beta-Extreme Value Distribution

Zafar Iqbal
Lecturer, Department of statistics The Islamia University of Bahawalpur. Pakistan
E-mail: zafariqbal101@hotmail.com

Dr. M Aleem (Corresponding author)
Chairman, Department of statistics The Islamia University of Bahawalpur .Pakistan
E-mail: draleemiub@hotmail.com

Abstract

In this research paper The Beta Extreme Value Type (III) distribution which is developed by Zafar and Aleem (2007) is considered and parameters are estimated by using moments of the Beta-Extreme Value (Type III) Distribution when the parameters 'm' & 'n' are real and moments of the Beta-Extreme Value (Type III) Distribution when the parameters 'm' & 'n' are integers and then a Comparison between r^{th} moments about origin when parameters are 'm' & 'n' are real and when parameters are 'm' & 'n' are integers. At the end second method, method of Maximum Likelihood is used to estimate the unknown parameters of the Beta Extreme Value Type (III) distribution.

1. Introduction

1.1 Beta-Extreme Value (Type III) Distribution:

The Beta-Extreme Value (Type III) Distribution Zafar and Aleem (2007), its generalized class is by defined as

$$f(x;m,n,a,b,c)=\frac{1}{\beta(m,n)}\frac{a}{b-c}\left(\frac{x-c}{b-c}\right)^{a-1}\left[1-e^{-\left(\frac{x-c}{b-c}\right)^{a}}\right]^{m-1}\left[e^{-\left(\frac{x-c}{b-c}\right)^{a}}\right]^{n} \quad (1.1)$$

Where $c\le x\le\infty, a,b,c,m,n\ge 0$

The Beta-Extreme value type (III) distribution is one of the reliability models. This distribution is useful for analyzing reliability and survival patterns. This model is also useful in extreme situation such as hot weather, earthquakes and flooding.

2. Moments of the Beta-Extreme Value (Type III) Distribution when 'm' & 'n' real:

By definition the r_{th} moment about origin is given by

$$\mu_r^{'}=\int_0^{\infty}x^r.f(x)dx \quad (2.1)$$

From (1.1) when 'm' and 'n' have real values.

$$f(x;m,n,a,b,c)=\frac{\Gamma m}{\beta(m,n)}\frac{a}{(b-c)^a}\sum_{i=0}^{m-1}(x-c)^{a-1}\frac{(-1)^i e^{-i\left(\frac{x-c}{b-c}\right)^a}}{i!\Gamma(m-i)}\left[e^{-\left(\frac{x-c}{b-c}\right)^{a}}\right]^{n} \quad (2.2)$$

Using (2.2) in (2.1)

$$\mu_r^{'}=\frac{a}{\beta(m,n)}\frac{1}{(b-c)^a}\sum_{i=0}^{m-1}(-1)^i\frac{\Gamma m}{i!\Gamma(m-i)}\int_c^{\infty}x^r(x-c)^{a-1}e^{-(n+i)\left(\frac{x-c}{b-c}\right)^a}dx$$

$y=\left(\frac{x-c}{b-c}\right)^a$ When x=c, then y=0, When x=∞, then y=∞

$$y.(b-c)^a = (x-c)^a \qquad (x-c) = (b-c)y^{\frac{1}{a}} \qquad dx = (b-c)\frac{1}{a}y^{\frac{1}{a}-1}dy$$

$$\mu_r' = \frac{a}{\beta(m,n)}\frac{1}{(b-c)^a}\sum_{i=0}^{m-1}(-1)^i\frac{\Gamma m}{\Gamma(m-i)}\frac{1}{i!}\int_0^\infty\left(c+(b-c)y^{\frac{1}{a}}\right)^r\left((b-c)y^{\frac{1}{a}}\right)^{a-1}\left(e^{-(n+i)y}\right)(b-c)\frac{1}{a}y^{\frac{1}{a}-1}dy$$

$$\mu_r' = \frac{1}{\beta(m,n)}\frac{1}{(b-c)^a}\sum_{i=0}^{m-1}(-1)^i\frac{\Gamma m}{\Gamma(m-i)}\frac{1}{i!}\int_0^\infty\left(\sum_{j=0}^{r}\quad\right)^r\left((b-c)y^{\frac{1}{a}}\right)^{a-1}\left(e^{-(n+i)y}\right)(b-c)y^{\frac{1}{a}-1}dy$$

$$\mu_r' = \frac{1}{\beta(m,n)}\sum_{i=0}^{m-1}(-1)^i\frac{\Gamma m}{\Gamma m-i}\frac{1}{i!}\int\sum_{j=0}^{r}\binom{r}{j}(C)^j(b-c)^{r-j+a-1+1-a}(y)^{\frac{r-j}{a}+1-\frac{1}{a}+\frac{1}{a}-1}\left(e^{-(n+i)y}\right)dy \tag{2.3}$$

We know that

$$(y)^{\frac{r-j}{a}}\left(e^{-(n+i)y}\right)dy = \Gamma\left(\frac{1-j}{a}+1\right)$$

So (2.3) becomes

$$\mu_r' = D\sum_{j=0}^{r}\binom{r}{j}(C)^j(b-c)^{r-j}\frac{\Gamma\left(\frac{1-j}{a}+1\right)}{(n+i)^{\frac{r-j}{a}+1}} \tag{2.4}$$

Where $D = \frac{1}{\beta(m,n)}\sum(-1)^i\frac{\Gamma m}{i!\Gamma(m-i)}$

(2.4) gives the r_{th} moment of the Beta-Extreme Value (Type III) Distribution.

From (2.4) we obtained the first four moments about origin of Beta-Extreme value (type III) distribution.

When r=1 $$\mu_1' = D\sum_{j=0}^{1}\binom{1}{j}(C)^j(b-c)^{1-j}\frac{\Gamma\left(\frac{1-j}{a}+1\right)}{(n+i)^{\frac{1-j}{a}+1}}$$

When r=2 $$\mu_2' = D\sum_{j=0}^{2}\binom{2}{j}(C)^j(b-c)^{2-j}\frac{\Gamma\left(\frac{2-j}{a}+1\right)}{(n+i)^{\frac{2-j}{a}+1}}$$

When r=3 $$\mu_3' = D\sum_{j=0}^{3}\binom{3}{j}(C)^j(b-c)^{3-j}\frac{\Gamma\left(\frac{3-j}{a}+1\right)}{(n+i)^{\frac{3-j}{a}+1}}$$

When r=4 $$\mu_4^{'} = D\sum_{j=0}^{4}\binom{4}{j}(C)^j (b-c)^{4-j}\,\frac{\Gamma\left(\frac{4-j}{a}+1\right)}{(n+i)^{\frac{4-j}{a}+1}}$$

$$\mu_2 = \mu_2^{'} - (\mu_1^{'})^2 \qquad \mu_3 = \mu_3^{'} - 3\mu_2^{'}\mu_1^{'} + 2(\mu_2^{'})^3$$

$$\mu_4 = \mu_4^{'} - 3\mu_3^{'}\mu_1^{'} + 6\mu_2^{'}(\mu_1^{'})^2 - 3(\mu_1^{'})^4$$

3. Moments of the Beta-Extreme Value (Type III) Distribution when 'm' & 'n' integers:

By definition the rth moment about origin is given by

$$\mu_r^{'} = \int_0^{\infty} x^r . f(x)dx \tag{3.1}$$

By using (1.1) in (3.1), we get

$$\mu_r^{'} = \frac{a}{\beta(m,n)}\frac{1}{(b-c)^a}\sum_{j=0}^{r}(-1)^i C_{(m,i)}\frac{1}{i!}\int_c^{\infty} x^r (x-c)^{a-1} e^{-(n+i)\left(\frac{x-c}{b-c}\right)^a} dx$$

By simplifying we get

$$\mu_r^{'} = D\sum_{j=0}^{r}\binom{r}{j}(C)^j (b-c)^{r-j}\,\frac{\frac{r-j}{a}+1}{(n+i)^{\frac{r-j}{a}+1}} \tag{3.2}$$

Where $D = \frac{1}{\beta(m,n)}\sum(-1)^i C_{(m,i)}$

(3.2) gives the r_{th} moment of the Beta-Extreme Value (Type III) Distribution, when 'm' and 'n' are integers. From (3.2) we obtained the first four moments about origin of Beta-Extreme value (type III) distribution.

When r=1 $$\mu_1^{'} = D\sum_{j=0}^{1}\binom{1}{j}(C)^j (b-c)^{1-j}\,\frac{\frac{1-j}{a}+1}{(n+i)^{\frac{1-j}{a}+1}}$$

When r=2 $$\mu_2^{'} = D\sum_{j=0}^{2}\binom{2}{j}(C)^j (b-c)^{2-j}\,\frac{\frac{2-j}{a}+1}{(n+i)^{\frac{2-j}{a}+1}}$$

When r=3 $$\mu_3^{'} = D\sum_{j=0}^{3}\binom{3}{j}(C)^j(b-c)^{3-j}\frac{\frac{3-j}{a}+1}{(n+i)^{\frac{3-j}{a}+1}}$$

When r=4 $$\mu_4^{'} = D\sum_{j=0}^{4}\binom{4}{j}(C)^j(b-c)^{4-j}\frac{\frac{4-j}{a}+1}{(n+i)^{\frac{4-j}{a}+1}}$$

$$\mu_2 = \mu_2^{'} - (\mu_1^{'})^2$$

$$\mu_3 = \mu_3^{'} - 3\mu_2^{'}\mu_1^{'} + 2(\mu_2^{'})^3$$

$$\mu_4 = \mu_4^{'} - 3\mu_3^{'}\mu_1^{'} + 6\mu_2^{'}(\mu_1^{'})^2 - 3(\mu_1^{'})^4$$

4. Comparison between r_{th} moments about origin when parameters 'm' & 'n' are real and parameters 'm' & 'n' are integers.

Case1: When 'c'=0

R_{th} moment for real values	R_{th} moment for integers values
$\mu_1^{'} = \frac{(b)(\Gamma m)\Gamma\left(\frac{1+a}{a}\right)}{\beta(m,n)}\sum_{i=0}^{m-1}(-1)^i\frac{1}{i!(n+i)^{\frac{1+a}{a}}\Gamma(m-i)}$	$\mu_1^{'} = \frac{(b)\Gamma\left(\frac{1+a}{a}\right)}{\beta(m,n)}\sum_{i=0}^{m-1}(-1)^i(C_{m,i})\frac{1}{(n+i)^{\frac{1+a}{a}}}$
$\mu_2^{'} = \frac{(b^2)(\Gamma m)\Gamma\left(\frac{2+a}{a}\right)}{\beta(m,n)}\sum_{i=0}^{m-1}(-1)^i\frac{1}{i!(n+i)^{\frac{2+a}{a}}\Gamma(m-i)}$	$\mu_2^{'} = \frac{(b^2)\Gamma\left(\frac{2+a}{a}\right)}{\beta(m,n)}\sum_{i=0}^{m-1}(-1)^i(C_{m,i})\frac{1}{(n+i)^{\frac{2+a}{a}}}$
$\mu_3^{'} = \frac{(b^3)(\Gamma m)\Gamma\left(\frac{3+a}{a}\right)}{\beta(m,n)}\sum_{i=0}^{m-1}(-1)^i\frac{1}{i!(n+i)^{\frac{3+a}{a}}\Gamma(m-i)}$	$\mu_3^{'} = \frac{(b^3)\Gamma\left(\frac{3+a}{a}\right)}{\beta(m,n)}\sum_{i=0}^{m-1}(-1)^i(C_{m,i})\frac{1}{(n+i)^{\frac{3+a}{a}}}$
$\mu_4^{'} = \frac{(b^4)(\Gamma m)\Gamma\left(\frac{4+a}{a}\right)}{\beta(m,n)}\sum_{i=0}^{m-1}(-1)^i\frac{1}{i!(n+i)^{\frac{4+a}{a}}\Gamma(m-i)}$	$\mu_4^{'} = \frac{(b^4)\Gamma\left(\frac{4+a}{a}\right)}{\beta(m,n)}\sum_{i=0}^{m-1}(-1)^i(C_{m,i})\frac{1}{(n+i)^{\frac{4+a}{a}}}$

Case 2: When 'm'=1 and 'c'=0 then for different values of parameters 'a', 'b' and 'n' the moments about origin becomes

R_{th} moment for real values	**R_{th} moment for integers values**
$\mu_1' = \frac{nb\Gamma\left(\frac{1+a}{a}\right)}{(n)^{\frac{1+a}{a}}}$	$\mu_1' = \frac{nb\Gamma\left(\frac{1+a}{a}\right)}{(n)^{\frac{1+a}{a}}}$
$\mu_2' = \frac{n(b^2)\Gamma\left(\frac{2+a}{a}\right)}{(n)^{\frac{1+a}{a}}}$	$\mu_2' = \frac{n(b^2)\Gamma\left(\frac{2+a}{a}\right)}{(n)^{\frac{1+a}{a}}}$
$\mu_3' = \frac{n(b^3)\Gamma\left(\frac{3+a}{a}\right)}{(n)^{\frac{1+a}{a}}}$	$\mu_3' = \frac{n(b^3)\Gamma\left(\frac{3+a}{a}\right)}{(n)^{\frac{1+a}{a}}}$
$\mu_4' = \frac{n(b^4)\Gamma\left(\frac{4+a}{a}\right)}{(n)^{\frac{1+a}{a}}}$	$\mu_4' = \frac{n(b^4)\Gamma\left(\frac{4+a}{a}\right)}{(n)^{\frac{1+a}{a}}}$

Case 3: When 'n'=1 & 'c'=0 then for different values of parameters 'a' , 'b' and 'm' the moments about origin becomes

R_{th} moment for real values	**R_{th} moment for integers values**
$\mu_1' = (b\ )\Gamma(m+1)\Gamma\left(\frac{1+a}{a}\right)\sum_{i=0}^{m-1}(-1)^i \frac{1}{i!(1+i)^{\frac{1+a}{a}}\Gamma(m-i)}$	$\mu_1' = \frac{(b)\Gamma\left(\frac{1+a}{a}\right)}{\beta(m,1)}\sum_{i=0}^{m-1}(-1)^i(C_{m,i})\frac{1}{(n+i)^{\frac{1+a}{a}}}$
$\mu_2' = (b^2)(\Gamma(m+1)\Gamma\left(\frac{2+a}{a}\right)\sum_{i=0}^{m-1}(-1)^i \frac{1}{i!(1+i)^{\frac{2+a}{a}}\Gamma(m-i)}$	$\mu_2' = \frac{(b^2)\Gamma\left(\frac{2+a}{a}\right)}{\beta(m,1)}\sum_{i=0}^{m-1}(-1)^i(C_{m,i})\frac{1}{(n+i)^{\frac{2+a}{a}}}$
$\mu_3' = (b^3)\Gamma(m+1)\Gamma\left(\frac{3+a}{a}\right)\sum_{i=0}^{m-1}(-1)^i \frac{1}{i!(1+i)^{\frac{3+a}{a}}\Gamma(m-i)}$	$\mu_3' = \frac{(b^3)\Gamma\left(\frac{3+a}{a}\right)}{\beta(m,1)}\sum_{i=0}^{m-1}(-1)^i(C_{m,i})\frac{1}{(n+i)^{\frac{3+a}{a}}}$

$\mu_4^{'} = (b^4)\Gamma(m+1)\Gamma\left(\frac{4+a}{a}\right)\sum_{i=0}^{m-1} (-1)^i \frac{1}{i!(1+i)^{\frac{4+a}{a}}\Gamma(m-i)}$	$\mu_4^{'} = \frac{(b^4)\Gamma\left(\frac{4+a}{a}\right)}{\beta(m,1)}\sum_{i=0}^{m-1} (-1)^i (C_{m,i}) \frac{1}{(n+i)^{\frac{4+a}{a}}}$

Case 4: When a=1 & c=0 then for different values of parameters 'b' , 'n' and 'm' the moments about origin becomes

$\mathbf{R_{th}}$ **moment for real values**	$\mathbf{R_{th}}$ **moment for integers values**
$\mu_1^{'} = \frac{(b)(\Gamma m)}{\beta(m,n)}\sum_{i=0}^{m-1} (-1)^i \frac{1}{i!(n+i)^2\Gamma(m-i)}$	$\mu_1^{'} = \frac{(b)}{\beta(m,n)}\sum_{i=0}^{m-1} (-1)^i (C_{m,i}) \frac{1}{(n+i)^2}$
$\mu_2^{'} = \frac{2(b^2)(\Gamma m)}{\beta(m,n)}\sum_{i=0}^{m-1} (-1)^i \frac{1}{i!(n+i)^3\Gamma(m-i)}$	$\mu_2^{'} = \frac{2(b^2)}{\beta(m,n)}\sum_{i=0}^{m-1} (-1)^i (C_{m,i}) \frac{1}{(n+i)^3}$
$\mu_3^{'} = \frac{6(b^3)(\Gamma m)}{\beta(m,n)}\sum_{i=0}^{m-1} (-1)^i \frac{1}{i!(n+i)^4\Gamma(m-i)}$	$\mu_3^{'} = \frac{6(b^3)}{\beta(m,n)}\sum_{i=0}^{m-1} (-1)^i (C_{m,i}) \frac{1}{(n+i)^4}$
$\mu_4^{'} = \frac{24(b^4)(\Gamma m)}{\beta(m,n)}\sum_{i=0}^{m-1} (-1)^i \frac{1}{i!(n+i)^5\Gamma(m-i)}$	$\mu_4^{'} = \frac{24(b^4)}{\beta(m,n)}\sum_{i=0}^{m-1} (-1)^i (C_{m,i}) \frac{1}{(n+i)^5}$

Case 5: When b=1 & c=0 then for different values of parameters 'a' , 'n' and 'm' the moments about origin becomes

$\mathbf{R_{th}}$ **moment for real values**	$\mathbf{R_{th}}$ **moment for integers values**
$\mu_1^{'} = \frac{(\Gamma m)\Gamma\left(\frac{1+a}{a}\right)}{\beta(m,n)}\sum_{i=0}^{m-1} (-1)^i \frac{1}{i!(n+i)^{\frac{1+a}{a}}\Gamma(m-i)}$	$\mu_1^{'} = \frac{\Gamma\left(\frac{1+a}{a}\right)}{\beta(m,n)}\sum_{i=0}^{m-1} (-1)^i (C_{m,i}) \frac{1}{(n+i)^{\frac{1+a}{a}}}$
$\mu_2^{'} = \frac{(\Gamma m)\Gamma\left(\frac{2+a}{a}\right)}{\beta(m,n)}\sum_{i=0}^{m-1} (-1)^i \frac{1}{i!(n+i)^{\frac{2+a}{a}}\Gamma(m-i)}$	$\mu_2^{'} = \frac{\Gamma\left(\frac{2+a}{a}\right)}{\beta(m,n)}\sum_{i=0}^{m-1} (-1)^i (C_{m,i}) \frac{1}{(n+i)^{\frac{2+a}{a}}}$
$\mu_3^{'} = \frac{(\Gamma m)\Gamma\left(\frac{3+a}{a}\right)}{\beta(m,n)}\sum_{i=0}^{m-1} (-1)^i \frac{1}{i!(n+i)^{\frac{3+a}{a}}\Gamma(m-i)}$	$\mu_3^{'} = \frac{\Gamma\left(\frac{3+a}{a}\right)}{\beta(m,n)}\sum_{i=0}^{m-1} (-1)^i (C_{m,i}) \frac{1}{(n+i)^{\frac{3+a}{a}}}$

$\mu_4' = \dfrac{(\Gamma m)\Gamma\left(\dfrac{4+a}{a}\right)}{\beta(m,n)} \sum_{i=0}^{m-1} (-1)^i \dfrac{1}{i!(n+i)^{\frac{4+a}{a}} \Gamma(m-i)}$	$\mu_4' = \dfrac{\Gamma\left(\dfrac{4+a}{a}\right)}{\beta(m,n)} \sum_{i=0}^{m-1} (-1)^i (C_{m,i}) \dfrac{1}{(n+i)^{\frac{4+a}{a}}}$

5. Method of Maximum Likelihood:

We used method of Maximum Likelihood for the estimation of parameters of Beta-Extreme Value (Type III) Distribution.

We know that

$$f(x) = \frac{1}{\beta(m,n)} \frac{a}{b-c} \left[\frac{x-c}{b-c}\right]^{a-1} \left[1 - e^{-(\frac{x-c}{b-c})^a}\right]^{m-1} \left[e^{-(\frac{x-c}{b-c})^a}\right]^n$$

The maximum likelihood function of the above equation is.

$$L(x;m,n,a,b,c) = \prod_{i=1}^{n} \frac{1}{\beta(m,n)} \frac{a}{b-c} \left[\frac{x-c}{b-c}\right]^{a-1} \left[1 - e^{-(\frac{x-c}{b-c})^a}\right]^{m-1} \left[e^{-(\frac{x-c}{b-c})^a}\right]^n$$

$$L(x;m,n,a,b,c) = \left[\frac{1}{\beta(m,n)}\right]^n \left(\frac{a}{b-c}\right)^n \prod_{i=1}^{n} \left[\frac{x-c}{b-c}\right]^{a-1} \prod_{i=1}^{n} \left[1 - e^{-\left(\frac{x-c}{b-c}\right)^a}\right]^{m-1} \prod_{i=1}^{n} \left[e^{-\left(\frac{x-c}{b-c}\right)^a}\right]^n$$

By taking log on both sides

$$\log \mathrm{L}(x;m,n,a,b,c) = \mathrm{n}\left[\log(1) - \log\beta(\mathrm{m,n})\right] + n\left[\log a - \log(b-c)\right]$$

$$+ (a-1)\sum_{i=1}^{n} \log\left(\frac{x-c}{b-c}\right) + (m-1)\sum_{i=1}^{n} \log\left[1 - e^{-\left(\frac{x-c}{b-c}\right)^a}\right] - n\sum_{i=1}^{n}\left(\frac{x-c}{b-c}\right)^a$$

$$\log L(x;m,n,a,b,c) = -n\log\beta(m,n) + n\log a - n\log(b-c) + (a-1)\sum_{i=1}^{n}\log(\frac{x-c}{b-c})$$

$$+ (m-1)\sum_{i=1}^{n} \log\left(1 - e^{-\left(\frac{x-c}{b-c}\right)^a}\right) - n\sum_{i=1}^{n}\left(\frac{x-c}{b-c}\right)^a$$

$$\frac{\partial \log L}{\partial m} = -n\frac{\partial \log\beta(m,n)}{\partial m} + \sum_{i=1}^{n} \log\left[1 - e^{-\left(\frac{x-c}{b-c}\right)^a}\right]$$

$$= -n\frac{\partial \log\beta(m,n)}{\partial m} + \sum_{i=1}^{n}\log\left[1-e^{-\left(\frac{x-c}{b-c}\right)^{a}}\right]$$

$$\frac{\partial \log L}{\partial m} = -n\psi(m,n) + \sum_{i=1}^{n}\left[1-e^{-\left(\frac{x-c}{b-c}\right)^{a}}\right] \qquad (5.1)$$

$$\frac{\partial \log L}{\partial n} = -n\frac{\partial \beta(m,n)}{\partial n} - \log\beta(m,n) + \log a - \log(b-c) - \sum_{i=1}^{n}\left(\frac{x-c}{b-c}\right)^{a}$$

$$\frac{\partial \log L}{\partial n} = -n\Psi(\mathrm{m,n}) - \log\beta(\mathrm{m,n}) + \log \mathrm{a} - \log(\mathrm{b-c}) - \sum_{\mathrm{i=1}}^{\mathrm{n}}\left(\frac{x-c}{b-c}\right)^{a} \qquad (5.2)$$

$$\frac{\partial \log L}{\partial \mathrm{a}} = \frac{n}{b} + \sum_{i=1}^{n}\log\left(\frac{x-c}{b-c}\right) + (m-1)\sum_{i=1}^{n}\frac{1}{\left[1-e^{-\left(\frac{x-c}{b-c}\right)^{a}}\right]}\frac{\partial}{\partial \mathrm{a}}\left[1-e^{-\left(\frac{x-c}{b-c}\right)^{a}}\right]$$

$$-n\sum_{i=1}^{n}\left(\frac{x-c}{b-c}\right)^{a}\log\left(\frac{x-c}{b-c}\right) \qquad (5.3)$$

$$\frac{\partial \log L}{\partial b} = -\frac{n}{b-c} - (a-1)\sum_{i=1}^{n}\frac{1}{b-c} + (m-1)\sum_{i=1}^{n}\frac{1}{1-e^{-(\frac{x-c}{b-c})^{a}}}\frac{\partial}{\partial \mathrm{b}}\left[1-e^{-\left(\frac{x-c}{b-c}\right)^{a}}\right]$$

$$-na\sum_{i=1}^{n}\left(\frac{x-c}{b-c}\right)^{a-1}\frac{\partial}{\partial \mathrm{b}}\left(\frac{x-c}{b-c}\right)$$

$$= \frac{-n}{b-c} - (a-1)\sum_{i=1}^{n}\frac{1}{b-c} + (m-1)\sum_{i=1}^{n}\left[\frac{-e^{-\left(\frac{x-c}{b-c}\right)^{a}}}{1-e^{-\left(\frac{x-c}{b-c}\right)^{a}}} - a\left[\frac{x-c}{b-c}\right]^{a-1}\frac{\partial}{\partial b}\left(\frac{x-c}{b-c}\right)\right]$$

$$-na\sum_{i=1}^{n}\left(\frac{x-c}{b-c}\right)^{a-1}\left[\frac{-(x-c)}{(b-c)^{2}}\right]$$

$$\frac{\partial \log L}{\partial b} = \frac{-n}{b-c} - \frac{n(a-1)}{b-c} + a(m-1)\sum_{i=1}^{n} \frac{e^{-(\frac{x-c}{b-c})^a}}{1-e^{-(\frac{x-c}{b-c})^a}} \left[\frac{x-c}{b-c}\right]^{a-1} \left[\frac{-(x-c)}{(b-c)^2}\right]$$

$$+ na\sum_{i=1}^{n} \left(\frac{x-c}{b-c}\right)^{a-1} \frac{x-c}{(b-c)^2}$$

$$= \frac{-n-na+n}{(b-c)} - a(m-1)\sum_{i=1}^{n} \frac{e^{-(\frac{x-c}{b-c})^a}}{1-e^{-(\frac{x-c}{b-c})^a}} \left[\frac{x-c}{b-c}\right]^{a-1} \frac{x-c}{(b-c)^L}$$

$$+ na\sum_{i=1}^{n} \left(\frac{x-c}{b-c}\right)^{a-1} \left(\frac{x-c}{(b-c)^2}\right)$$

$$\frac{\partial \log L}{\partial b} = \frac{-na}{(b-c)} + a(m-1)\sum_{i=1}^{n} \frac{e^{-(\frac{x-c}{b-c})^a}}{1-e^{-(\frac{x-c}{b-c})^a}} \left[\frac{x-c}{b-c}\right]^{a-1} \frac{x-c}{(b-c)^L}$$

$$+ na\sum_{i=1}^{n} \left(\frac{x-c}{b-c}\right)^{a-1} \left(\frac{x-c}{(b-c)^2}\right) \qquad (5.4)$$

$$\frac{\partial \log L}{\partial c} = +\frac{n}{b-c} + (a-1)\sum_{i=1}^{n} \frac{1}{\left(\frac{x-c}{b-c}\right)} \frac{\partial}{\partial c}\left(\frac{x-c}{b-c}\right) + (m-1)\sum_{i=1}^{n} \frac{1}{1-e^{-\left(\frac{x-c}{b-c}\right)^a}} \frac{\partial}{\partial \mathrm{c}}\left[1-e^{-\left(\frac{x-c}{b-c}\right)^a}\right]$$

$$= \frac{-n}{b-c} + (a-1)(b-c)\sum_{i=1}^{n} (x-c)^{-1}\left[\frac{(b-c)(a-1)+(x-c)}{(b-c)^2}\right]$$

$$+ (m-1)\sum_{i=1}^{n} \frac{1}{1-e^{-\left(\frac{x-c}{b-c}\right)^a}} \left[-e^{-\left(\frac{x-c}{b-c}\right)^a} - a\left(\frac{x-c}{b-c}\right)^{a-1}\right]\left[\frac{-(b-c)+(x-c)}{(b-c)^2}\right]$$

$$= \frac{-n}{b-c} + (a-1)\sum_{i=1}^{n} \frac{-(b-c)+(x-c)}{(b-c)}(x-c)^{-1}$$

$$+(m-1)\sum_{i=1}^{n}\frac{1}{1-e^{-\left(\frac{x-c}{b-c}\right)^{a}}}\left[-a\,e^{-\left(\frac{x-c}{b-c}\right)^{a}}\left(\frac{x-c}{b-c}\right)^{a-1}-\frac{(b-c)+(x-c)}{(b-c)^{2}}\right]$$

$$\frac{\partial \log L}{\partial c}=\frac{-n}{b-c}+\frac{a-1}{b-c}\sum_{i=1}^{n}\left(\frac{x-b}{x-c}\right)\frac{-a(m-1)}{(b-c)^{a+1}}\sum_{i=1}^{n}\left[1-e^{-\left(\frac{x-c}{b-c}\right)^{a}}\right]^{-1}e^{-\left(\frac{x-c}{b-c}\right)^{a}}.(x-c)^{a-1}(x-b) \qquad (5.5)$$

Where $\Psi(\mathrm{x})=\partial \ln\Gamma x/\partial x$ is the derivative of the gamma function and equating these expressions (5.1), (5.2), (5.3), (5.4), (5.5) equal to zero then solving them simultaneously yields the maximum likelihood estimates of the five parameters Beta-Extreme Value (Type III) Distribution.

Equations (5.1), (5.2), (5.3), (5.4), (5.5) are solved iteratively to obtain $\hat{m},\hat{n},\hat{a},\hat{b}$ and $\hat{c}$ the maximum likelihood estimates (MLE) for parameters m,n,a,b and c respectively. By taking the second partial derivatives of (5.1), (5.2), (5.3), (5.4), (5.5) and the fisher information matrix can be obtained by taking the expectations of second partial derivatives with minus sign. The Inverse of the fisher information matrix provides the variances and the covariances for the maximum likelihood estimators.

Biblography

Memon, A. G. and Shaikh, N.M. (2003). Fitting Gumbel and Generalized Extreme Value Probability Distributions on Floods data of Indus River at Sukkur Barrage, [unpublished report].

Costantino, R.F., and Desharnais, R.A. (1981). Gamma Distributions of Adult Numbers for Tribolium Populations in the Region of Their Steady States. *Journal of Animal Ecology,* 50, 667–681.

Cox, D. R. and Oakes, D. (1984). *Analysis of Survival Data,* Chapman & Hall. London, UK.

Eugene, L. and Famoye, (2002). Beta-Normal Distribution And Its Application, *Communication is Statistics, -Theory Meth*., 31(4), 497-512.

Gupta, A.K. and Nadarajah, S. (2004). On the Moments of the Beta-Normal Distribution. *Communication I Statistics: Theory and Methods,* 33(1), 1-13.

Gupta, A.K., Nadarajah. S. (2004). *Handbook of Beta Distribution and its Applications*. Marcel Dekker Inc, New York.

Good, I.J. (1953). The Population Frequencies of the Species and the Estimation of Population Parameters. *Biometrika*. 40, 237-260.

Iqbal, Z. (2007). Beta-Extreme Value Distribution. M.Phil Thesis [Unpublished], The Islamia University Of Bahawalpur, Pakistan.

Holmes, J.D. and Moriarty, W.W. (1999). Application of the generalized Pareto distribution to extreme value analysis in wind engineering. *Journal of Wind Engineering and Industrial Aerodynamics*, 83(1-3), 1-10.

Lee, C,. Famoye. P. and Ohmolade, O. (2006). The Beta-Weibull Distribution: Some Properties and Applications to cencersord data.

Lock, M. O. (1973). *Reliability, Maintainability, and Availability Assessment*. Hayden Book Co. New Jersey, [ISBN: 0-87671-173-5]

Nadarajah. S., Kotz. S. (2004). *The Beta Gunmbel Distribution: Mathematical Problems in Engineering,* 323-332.

Nadarajah. S., Kotz. S. (2005). *The Beta Gunmbel Distribution: Reliability Engineering and System Safety*. 689-697.

Nelson, W. (1982). *Applied Life Data Analysis*. J. Wiley, New York. USA, [ISBN: 0-471-09458-7].

Eugene, N., Lee, C. and Famoye, F. (2002). Beta-Normal distribution and its applications. *Comm. Statist. Theory Methods,* 4, 497-512.

Sara, L, Nadrajah and Samuel, K. (2004). *The Beta Gumbel Distribution, Journal of mathematical problems in Engineering*. 4, 323-332.

Hanook, S. (2006). Three Parameter Beta-Inverse Weibull Distribution. M. Phil Thesis [Unpublished] Department of Statistics, G.C. University Lahore, Pakistan.

Kotz, S. and Nadarajah, S. (2000). *Extreme Value Distributions, Theory and Applications*. Imperial College Press, London.

Web Resources

http://www.itl.nist.gov/div898/handbook/apr/section1/apr163.htm

http://en.wikipedia.org/wiki/Generalized_extreme_value_distribution

http://en.wikipedia.org/wiki/Beta_distribution