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An Overview on Method of Cyclic Shifts for the Construction of Experimental Designs Useful in Business and Commerce

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Abstract

The competitive nature of business today requires the knowledge to be able to manipulate process levels in a predictable fashion and to reduce process variation (quality control). This goal can be achieved through the use of experimentation to identify and confirm the effects of the process variables. Therefore, design of experiments should be an integral part of process management. Experimental designs such as treatment balanced, neighbor balanced and balanced repeated measurements designs, have been extensively used in different fields, especially in business and commerce. Different methods have been used for their construction but method of cyclic shifts is the easiest one. This paper provides the overview, how this method has been used for the construction of the useful designs.

Keywords: cyclic shifts; experimental designs; competitive nature of business; polygonal designs; process management; quality control.

1. Introduction

Method of cyclic shifts (MOCS) has been widely used to construct different designs. This method was developed by Iqbal (1991). In this method, we can study some standard characteristics and properties of the specific design only through the set(s) of shifts. This method is quite simple to construct several types of important designs. Some of these are balanced incomplete block designs (BIBDs), partially BIBDs, polygonal designs (PDs), neighbor balanced designs (NBDs) and repeated measurements designs (RMDs). In this article, an overview is provided that how the MOCS is used to construct several types of important designs. This method consists of Rule I & Rule II.

According to Sanders et al. (2002), the competitive nature of business today requires the knowledge to be able to manipulate process levels in a predictable fashion and to reduce process variation. The need of this knowledge (the continuous improvement of processes) must be quicker than ever before. This goal requires the ongoing use of experimentation to identify and confirm the effects of the process variables. Therefore, designed experiments should be viewed as an integral part of process management. Whereas the statistical design of experiments (DOE) is a valuable tool for rapidly accumulating this process knowledge. In industrial experimentation, blocks are frequently a period of time when extraneous variables (variables not explicitly manipulated in the experiment) can reasonably be expected to remain constant while an experiment takes place. Periods of time (blocks) for subsequent experiments are selected so that while variables not explicitly manipulated can reasonably be expected to remain constant during the execution of a DOE, some might have changed between experiments (blocks). Polygonal designs are useful for survey and marketing.

2. Construction of BIBDs and PBIBDs

BIBDs and PBIBDs have the most significant role in experimental designs to compare each pair of treatments with equal or almost equal precision. Such experimental designs provide the assurance that treatments can be compared with same precision. Using method of cyclic shifts, Yasmin *et al.* (2015) and Jamil *et al.* (2017) presented BIBD and PBIBD respectively.

2.1 How to Obtain a BIBD and PBIBD Using Rule I.

MOCS is described in this Section only for BIBDs and PBIBDs.

Let $\underline{S}_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}]$, $1 \le q_{ji} \le v-1$. A design will be BIBD if each of 1, 2, …, v-1 appears λ times in S_j*. $\underline{S}_j^* = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}, (q_{j1}+q_{j2}), (q_{j2}+ q_{j3}), \ldots, (q_{j(k-2)}+q_{j(k-1)}), (q_{j1}+q_{j2}+ q_{j3}), (q_{j2}+q_{j3}+q_{j4}), \ldots, (q_{j(k-3)}+q_{j(k-2)}+q_{j(k-1)}), \ldots, (q_{j1}+q_{j2}+\ldots+ q_{j(k-1)})$, and complement of each element], here v-q_i is complement of q_i. It will be PBIBD (two associate) if λ has two values $\lambda_2 = \lambda_1+1$.

Example 2.1: S = [3, 1, 2, 2, 1, 1, 1] provide BIBD for v = 15, k = 8 in 15 blocks through MOCS (Rule I).

Here

S^* = [3,1,2,2,1,1,1,4,3,4,3,2,2,6,5,5,4,3,8,6,6,5,9,7,7,10,8,11,12,14,13,13,14,14,14,11, 12,11,12,13,13,9,10,10,11,12,7,9,9,10,6,8,8,5,7,4]. Here each element from 1, 2, 3, …, 14 appears 4 times. Hence it is BIBD with $\lambda = 4$.

Blocks														
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	4	5	6	7	8	9	10	11	12	13	14	0	1	2
4	5	6	7	8	9	10	11	12	13	14	0	1	2	3
6	7	8	9	10	11	12	13	14	0	1	2	3	4	5
8	9	10	11	12	13	14	0	1	2	3	4	5	6	7
9	10	11	12	13	14	0	1	2	3	4	5	6	7	8
10	11	12	13	14	0	1	2	3	4	5	6	7	8	9
11	12	13	14	0	1	2	3	4	5	6	7	8	9	10

2.2 How to Obtain a BIBD and PBIBD Using Rule II

Let $\underline{S}_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-2)}]t$, $1 \le q_{ji} \le v\text{-}2$. A design will be BIBD if each of 1, 2, …, v-2 appears λ times in S_j*. Where $\underline{S}_j^* = [q_{j1}, q_{j2}, \ldots, q_{j(k-2)}, (q_{j1}+q_{j2}), (q_{j2}+q_{j3}), \ldots, (q_{j(k-3)}+q_{j(k-2)}), (q_{j1}+q_{j2}+q_{j3}), (q_{j2}+q_{j3}+q_{j4}), \ldots, (q_{j(k-4)}+q_{j(k-3)}+q_{j(k-2)}), \ldots, (q_{j1}+q_{j2}+\ldots+ q_{j(k-2)})]$, here, v-1-q_i is complement of q_i. If λ has two values $\lambda_2 = \lambda_1+1$ then it is PBIB design with 2-association scheme.

Example 2.2: BIBD is constructed from the sets of shifts [1,2,3] and [1,4]t for $v = 8$, k = 4 in 14 blocks through Rule II. Here S^* = [1,2,3,3,5,6,1,4,5,6,5,4,4,2,1,6,3,2] Here each element from 1, 2, 3, …, 6 appears 3 times. Hence it is BIBD with $\lambda = 3$.

Blocks													
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
0	1	2	3	4	5	6	0	1	2	3	4	5	6
1	2	3	4	5	6	0	1	2	3	4	5	6	0
3	4	5	6	0	1	2	5	6	0	1	2	3	4
6	0	1	2	3	4	5	7	7	7	7	7	7	7

3. Construction of Polygonal Designs

PDs are important in survey sampling. Using MOCS, Tahir *et al.* (2011) constructed PDs for $k = 3$ with $\lambda = 1, 2, 3, 4, 6, 12$ and $\alpha = 2$, Intizar *et al.* (2016) constructed PDs in blocks of two different sizes 4 and 2. Following is the procedure to obtain PDs.

3.1 How to Obtain Polygonal Designs Using Rule I

Let $\underline{S}_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}]$, where $1 \leq q_{ji} \leq v-1$.

- If each of 2, …, v-2 appears λ times in S* except 1 and v-1 which do not appear then this design is CPD with $\alpha = 1$.
- If each of 3, …, v-3 appears λ times in S* except 1, 2, v-1 and v-2 which do not appear then this design is CPD with $\alpha = 2$.
- If each of $\alpha+1$, …, $v-(\alpha+1)$ appears λ times in S* except 1, 2,…, α, v-1, v-2,…, $v-\alpha$ which do not appear then this design is CPD with joint distance α.

Here, S* contains (i) each shift of **S**, (ii) sum (mod v) of each of two, three, …, k-1 consecutive shifts, and (iii) complement of each element in (i) & (ii). Here, $v-q_i$ is complement of q_i.

For a circular polygonal design with $\alpha = 1$, the concurrence matrix is:

$$NN' = \begin{pmatrix} r & 0 & \lambda & \lambda & \cdots & \lambda & 0 \\ 0 & r & 0 & \lambda & \cdots & \lambda & \lambda \\ \lambda & 0 & r & 0 & \cdots & \lambda & \lambda \\ \lambda & \lambda & 0 & r & \cdots & \lambda & \lambda \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda & \lambda & \lambda & \lambda & \cdots & r & 0 \\ 0 & \lambda & \lambda & \lambda & \cdots & 0 & r \end{pmatrix}$$

For a circular polygonal design with joint distance $\alpha = 2$ the concurrence matrix is

$$NN' = \begin{pmatrix} r & 0 & 0 & \lambda & \cdots & \lambda & 0 & 0 \\ 0 & r & 0 & 0 & \cdots & \lambda & \lambda & 0 \\ 0 & 0 & r & 0 & \cdots & \lambda & \lambda & \lambda \\ \lambda & 0 & 0 & r & \cdots & \lambda & \lambda & \lambda \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \lambda & \lambda & \lambda & \ldots & \cdots & r & 0 & 0 \\ 0 & \lambda & \lambda & \lambda & \cdots & 0 & r & 0 \\ 0 & 0 & \lambda & \lambda & \cdots & 0 & 0 & r \end{pmatrix}$$

Example 3.1: A circular polygonal design for $v = 11$, $k_1 = 3$, $k_2 = 2$, $\lambda = 1$ & $\alpha = 1$ is constructed using [2, 3]+[4]. The required design is:

Blocks										
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
0	1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	0	1
5	6	7	8	9	10	0	1	2	3	4
Blocks										
12	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
0	1	2	3	4	5	6	7	8	9	10
4	5	6	7	8	9	10	0	1	2	3

3.2 How to Obtain Polygonal Designs Using Rule II

Let $\underline{S}_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-2)}]t$, where $1 \le q_{ji} \le v\text{-}1$.

- If each of 2, …, v-3 appears λ times in S* except 1 and v-2 which do not appear then this design is CPD with α = 1.
- If each of 3, …, v-4 appears λ times in S* except 1, 2, v-2 and v-3 which do not appear then this design is CPD with α = 2.
- If each of α+1, …, v-1-(α+1) appears λ times in S* except 1, 2,…, α, v-2,…, v-1-α which do not appear then this design is CPD with joint distance α.

Example 3.2: [2,3]+[4]t provide PD for $v = 8$, k = 3, $\lambda = 2$ and $\alpha = 1$.

Here $S^* = [2,3,5,4,5,4,2,3]$ Here each element from 2, 3, …, 5 appears exactly twice. Hence it is polygonal design with λ = 2 and $\alpha = 1$.

Blocks													
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
0	1	2	3	4	5	6	0	1	2	3	4	5	6
2	3	4	5	6	0	1	4	5	6	0	1	2	3
5	6	0	1	2	3	4	7	7	7	7	7	7	7

4. Construction of Balanced and Strongly Balanced RMDs

RMDs have application in many branches such as business, commerce, agriculture, food science, animal husbandry, biology, education, psychology, and pharmacology. In the following literature, MOCS is used to construct RMDs.

Iqbal and Jones (1994), Iqbal and Tahir (2009), Iqbal et al. (2010), Bashir et al. (2018), Rajab et al. (2018), Rasheed et al. (2018), Ahmed et al. (2018), Khan et al. (2019), Daniyal et al. (2019), Rasheed et al. (2019), and Ahmed et al. (2019) constructed CBRMDs & CSBRMDs in periods of equal and unequal sizes. Jabeen et al. (2019), Nazeer et al. (2019), Jabeen et al. (2019) constructed minimal CSPBRMDs in periods of equal and two different sizes. Hussain et al. (2020) constructed CWBRMDs in two different periods sizes. Abdullah et al. (2019) constructed such designs in non-circular periods. Following is the to obtain the BRMDs.

4.1 How to Obtain BRMDs in Circular Periods using Rule I

Let S = [q_1, q_2, …, q_{p-1}], where 1 ≤ q_i ≤ v-1. If each of 1, 2, …, v-1 appears λ′ in S* then it be CBRMD in periods of size p, where S* = [q_1, q_2, …, q_{p-1}, v - (q_1+ q_2+ …+ q_{p-1}) mod v]. Design will be non-binary if sum of any two, three, …, or (p-1) consecutive elements of S is 0 (mod v), otherwise binary.

Example 4.1: Set of shifts [1,2,8]+[3,4,6] produce non-binary CBRMD for v = 9 and p = 4.

Here S^* = [2,1,8,7,3,4,6,5] Here each element from 1,2,3,…,8 appears once. Therefore, it is minimal CBRMDs with λ′ = 1.

Periods	Subjects								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	0_2	1_3	2_4	3_5	4_6	5_7	6_8	7_0	8_1
2	1_0	2_1	3_2	4_3	5_4	6_5	7_6	8_7	0_8
3	3_1	4_2	5_3	6_4	7_5	8_6	0_7	1_8	2_0
4	2_3	3_4	4_5	5_6	6_7	7_8	8_0	0_1	1_2

Periods	Subjects								
	10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
1	0_4	1_5	2_6	3_7	4_8	5_0	6_1	7_2	8_3
2	3_0	4_1	5_2	6_3	7_4	8_5	0_6	1_7	2_8
3	7_3	8_4	0_5	1_6	2_7	3_8	4_0	5_1	6_2
4	4_7	5_8	6_0	7_1	8_2	0_3	1_4	2_5	3_6

4.2 How to Obtained BRMDs in Circular Blocks Using Rule II

Let S_1 = [q_{11}, q_{12}, …, q_{p_1-1}] and S_2 = [q_{21}, q_{22}, …,q_{p_2-2}]t, where 1 ≤ q_{ij} ≤ v-2. If each of 1, 2, …, v-2 appears λ′ in S* then it is CBRMD in periods of size p, where S* = [q_{11}, q_{12}, …, q_{p_1-1}, (v-1)-{(q_{11}+ q_{12}+ …+ q_{p_1-1}) mod(v-1)}, q_{21}, q_{22}, …,q_{p_2-2}].

Example 4.2: Sets of shifts [2,1,6]+[3,4]t provide following non-binary CBRMD for v = 8, k = 4.

Here S^* = [1,2,6,5,3,4] and each element from 1, 2, 3, …, 6 appears exactly once. Hence it is minimal CBRMDs with λ′ = 1.

Periods	**Subjects**													
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	0_2	1_3	2_4	3_5	4_6	5_0	6_1	0_7	1_7	2_7	3_7	4_7	5_7	6_7
2	2_0	3_1	4_2	5_3	6_4	0_5	1_6	3_0	4_1	5_2	6_3	0_4	1_5	2_6
3	3_2	4_3	5_4	6_5	0_6	1_0	2_1	0_3	1_4	2_5	3_6	4_0	5_1	6_2
4	2_3	3_4	4_5	5_6	6_0	0_1	1_2	7_0	7_1	7_2	7_3	7_4	7_5	7_6

4.3 How to Obtain a CSBRMD Using Rule I

Let S_1 = [q_{11}, q_{12}, …,$q_{1(p-1)}$], where $0 \le q_{ij} \le v$-1. If each of 0, 1, 2, …, v-1 appears λ′ S* then it is CSBRMDs in period of size p, where S* = [q_{11}, q_{12}, …, $q_{1(p-1)}$, v-(q_{11}+ q_{12}+ …+ $q_{1(p-1)}$) mod v]. It will be minimal if λ′ = 1.

Example 4.3: [1,3,2,4,6,5] provides following CSBRMDs for v = 7 and p = 7.

Here S^* = [1,3,2,4,6,5,0] Here each element from 0,1,2,3,…,6 appears exactly once. Hence it is minimal CSBRMDs with λ′ = 1 using rule I.

Periods	**Subjects**						
	1	**2**	**3**	**4**	**5**	**6**	**7**
1	0_0	1_1	2_2	3_3	4_4	5_5	6_6
2	1_0	2_1	3_2	4_3	5_4	6_5	0_6
3	4_1	5_2	6_3	0_4	1_5	2_6	3_0
4	6_4	0_5	1_6	2_0	3_1	4_2	5_3
5	3_6	4_0	5_1	6_2	0_3	1_4	2_5
6	2_3	3_4	4_5	5_6	6_0	0_1	1_2
7	0_2	1_3	2_4	3_5	4_6	5_0	6_1

4.4 How to Obtain BRMDs in Linear Periods Using Rule I

Let S = [q_1, q_2, …, q_{p-1}], where $1 \le q_i \le v$-1. If each of 1, 2, …, v-1 appears λ′ in S* then it is BRMD in linear periods of size p, where S* = [q_1, q_2, …, q_{p-1}].

Example 4.4: Sets of shifts [2, 1, 3,4]+[5, 6, 8, 7] provide following CBRMDs in linear periods for v = 9 in p = 5.

Here S^* = [2,1,3,4,5,6,8,7] and each element from 1, 2, 3,…, 8 appears once, therefore, it is minimal BRMDs with λ′ = 1 in linear periods, using rule I.

Periods	**Subjects**																	
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
1	0	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8
2	2_0	3_1	4_2	5_3	6_4	7_5	8_6	0_7	1_8	5_0	6_1	7_2	8_3	0_4	1_5	2_6	3_7	4_8
3	3_2	4_3	5_4	6_5	7_6	8_7	0_8	1_0	2_1	2_5	3_6	4_7	5_8	6_0	7_1	8_2	0_3	1_4
4	6_3	7_4	0_5	1_6	2_7	3_8	4_0	5_1	6_2	1_2	2_3	3_4	4_5	5_6	6_7	7_8	8_0	0_1
5	2_6	3_7	4_0	5_1	6_2	7_3	0_4	1_5	0_6	8_1	0_2	1_3	2_4	3_5	4_6	5_7	6_8	7_0

4.5 How to Obtain BRMDs in Linear Periods Using Rule II

Let $S_j = [q_{j1}, q_{j2}, \ldots, q_{j(p-1)}]$ and $S_i = [q_{i1}, q_{i2}, \ldots, q_{i(p-2)}]$t, where $1 \leq q_{ij} \leq v-2$. If each of 1, 2,…, v-2 appears λ′ in S* then it is non-linear BRMD in periods of size p, where $S^* = [q_{j1}, q_{j2}, \ldots, q_{j(p-1)}, q_{i1}, q_{i2}, \ldots, q_{i(p-2)}] \bmod (v-1)$.

Example 4.5: Sets of shifts [1,2,3,4]+[5,6,7]t provide following CBRMDs for v = 9 & p = 5 in linear periods, using Rule II.

Here S^* = [1,2,3,4,5,6,7] and each element from 0, 1, 2, 3, …, 7 appears exactly once. Hence it is minimal BRMDs in linear blocks with λ′ = 1, using Rule II.

Periods	**Subjects**																	
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
1	0	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8
2	1_0	2_1	3_2	4_3	5_4	6_5	7_6	8_7	0_8	5_0	6_1	7_2	8_3	0_4	1_5	2_6	3_7	4_8
3	3_1	4_2	5_3	6_4	7_5	8_6	0_7	1_8	2_0	2_5	3_6	4_7	5_8	6_0	7_1	8_2	0_3	1_4
4	6_3	7_4	8_5	0_6	1_7	2_8	3_0	4_1	5_2	0_2	1_3	2_4	3_5	4_6	5_7	6_8	7_0	8_1
5	1_6	2_7	3_8	4_0	5_1	6_2	7_3	8_4	0_5	8_0	8_1	8_2	8_3	8_4	8_5	8_6	8_7	8_8

4.6 How to Obtain SBRMD in Linear Periods Using Rule I

Let S = $[q_1, q_2, \ldots, q_{p-1}]$, where $0 \leq q_i \leq v-1$. If each of 1, 2, …, v-1 appears λ′ in S* then it is BRMD in linear periods of size p, where S* = $[q_1, q_2, \ldots, q_{p-1}]$.

Example 4.6: Set of shifts [1,2,3,0]+[4,5,6,7] produce SBRMD for v = 8 & p = 5 in linear periods.

Here S^* = [1,2,3,0,4,5,6,7] Here each element from 0,1,2,3,…,7 appears exactly once. Hence it is minimal SBRMDs in linear blocks with $\lambda' = 1$, using Rule I.

Periods	**Subjects**															
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	0_6	1_7	2_0	3_1	4_2	5_3	6_4	7_5	0_6	1_7	2_0	3_1	4_2	5_3	6_4	7_5
2	1_0	2_1	3_2	4_3	5_4	6_5	7_6	0_7	4_0	5_1	6_2	7_3	0_4	1_5	2_6	3_7
3	3_1	4_2	5_3	6_4	7_5	0_6	1_7	2_0	1_4	2_5	3_6	4_7	5_0	6_1	7_2	0_3
4	6_3	7_4	0_5	1_6	2_7	3_0	4_1	5_2	7_1	0_2	1_3	2_4	3_5	4_6	5_7	6_0
5	6_6	7_7	0_0	1_1	2_2	3_3	4_4	5_5	6_7	7_0	0_1	1_2	2_3	3_4	4_5	5_6

5. Construction of Neighbor Balanced Design

NBDs are used to control neighbor effects. NBDs constructed through MOCS up to 2011 can be found in Ahmed et al. (2011). Ahmed and Akhtar (2011), Ahmed and Akhtar (2012 a, b) Yasmin et al. (2013), Ahmed et al. (2013), Ahmed and Akhtar (2013), Yasmin et al. (2014), Ahmed et al. (2014), Ahmed and Akhtar (2015), Ahmed et al. (2016), Shahid et al. (2017), Ahmed et al. (2017), Khalid et al. (2018), and Shahid et al. (2019) constructed circular and non-circular NBDs. Following is the procedure to obtain NBDs.

5.1 How to Obtain NBDs in Linear Blocks Using Rule I

Let $\underline{S}_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}]$, where $1 \le q_{ji} \le v-1$. If each of 1, 2, …, v-1 appears λ' times in S* then design is NBD in linear blocks, where $S^* = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}, v-q_{j1}, v-q_{j2}, \ldots, v-q_{j(k-1)}]$.

Example 5.1: Sets of shifts [1,2,3]+[4,5,6] provide NBD for v = 13 in linear blocks of size 4 with $\lambda' = 1$.

Here S^* = [1,2,3,4,5,6,12,11,10,9,8,7] and each element from 1, 2, 3, …, 12 appears exactly once. Hence it is minimal NBD in linear blocks using Rule I.

Blocks												
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	0
3	4	5	6	7	8	9	10	11	12	0	1	2
6	7	8	9	10	11	12	0	1	2	3	4	5
Blocks												
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
0	1	2	3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	0	1	2	3
9	10	11	12	0	1	2	3	4	5	6	6	7
2	3	4	5	6	7	8	9	10	11	12	0	1

5.2 How to Obtain NBDs in Linear Blocks Using Rule II

Let $\underline{S}_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-2)}]t$, where $1 \leq q_{ji} \leq v\text{-}2$. If each of 1, 2, …, v-2 appears λ' times in S*, design is NBD in linear blocks, where $S^* = [q_{j1}, q_{j2}, \ldots, q_{j(k-2)}, v\text{-}1\text{-}q_{j1}, v\text{-}1\text{-}q_{j2}, \ldots, v\text{-}1\text{-}q_{j(k-2)}]$.

Example 5.2: [1,2]+[3]t produce minimal NBD for v = 8 & k = 3.

Here S^* = [1,2,3,6,5,4] and each element from 1, 2, 3, …, 6 appears exactly once. Hence it is minimal NBD in linear blocks using Rule II.

Blocks													
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
0	1	2	3	4	5	6	0	1	2	3	4	5	6
1	2	3	4	5	6	0	3	4	5	6	0	1	2
3	4	5	6	0	1	2	7	7	7	7	7	7	7

5.3 How to Obtain Nearest Neighbor Balanced Design Using Rule I

Rule I: Let $S = [q_1, q_2, \ldots, q_{k-1}]$, where $1 \leq q_i \leq v\text{-}1$. If each of 1, 2, …, v-1 appears λ' in S* then it is CNBD, where $S^* = [q_1, q_2, \ldots, q_{k-1}, (q_1+q_2+\ldots+q_{k-1}) \bmod v, v\text{-}(q_1), v\text{-}(q_2), \ldots, v\text{-}(q_{k-1}), v\text{-}(q_1+q_2+\ldots+q_{k-1}) \bmod v]$.

Example 5.3: Set of shifts [1,2,3,9,5] provides minimal NBD for v = 13 and k = 6 from.

Here S* = [1,2,3,9,5,7,6,8,4,10,11,12] and each element from 1, 2, 3, …, 12 appears once, therefore, it is minimal CNBDs, using Rule I.

Blocks												
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	0
3	4	5	6	7	8	9	10	11	12	0	1	2
6	7	8	9	10	11	12	0	1	2	3	4	5
2	3	4	5	6	7	8	9	10	11	12	0	1
7	8	9	10	11	12	0	1	2	3	4	5	6

5.4 How to Obtain Nearest Neighbor Balanced Design Using Rule II

Let S = [q_1, q_2, …, q_{k-2}]t, where $1 \leq q_i \leq v$-2. If each of 1, 2, …, v-2 appears λ' in S* then it is CNBD, where S* = [q_1, q_2, …, q_{k-1}, v-1-q_1, v-1-q_2, …, v-1-q_{k-2}].

Example 5.4: Sets of shifts [1, 1, 1, 1, 1, 1, 1, 1, 1, 1](1/11)+[2, 3, 4, 5, 5, 4, 3, 2, 1]t provide minimal NBD for v = 12 and k = 11.

Blocks											
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	0	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	0	1
2	5	6	7	8	9	10	0	1	2	3	4
3	9	10	0	1	2	3	4	5	6	7	8
4	3	4	5	6	7	8	9	10	0	1	2
5	8	9	10	0	1	2	3	4	5	6	7
6	1	2	3	4	5	6	7	8	9	10	0
7	4	5	6	7	8	9	10	0	1	2	3
8	6	7	8	9	10	0	1	2	3	4	5
9	7	8	9	10	0	1	2	3	4	5	6
10	11	11	11	11	11	11	11	11	11	11	11

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